

Mechatronic Systems with Uncertain Physical Parameters

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During the design of controllers for mechatronic systems the designer often has to deal with uncertain physical parameters. Quantitative Feedback Theory can handle this parametric uncertainty, but this design method for robust controllers requires the uncertainty to be specified in uncertainty regions in the Nichols chart. Several techniques to map uncertain physical parameters onto uncertainty regions in the Nichols chart will be discussed. For one technique computer-based support has been developed to enhance the automated design of QFT-controllers for mechatronic systems with uncertain physical parameters

1 INTRODUCTION

Controller design for mechatronic systems generally is based on a physical model of the electro-mechanical plant. In the conceptual design stage as well as during operation, the *physical parameters* in the model are not known exactly or may be time varying. Lower and upper bounds for these parameters can often be indicated, resulting in so-called structured uncertainty. In the physical model of Figure 1 the uncertain physical parameters are the stiffness c and the mass of the end-effector m_1 . This system is used as an example throughout this paper.

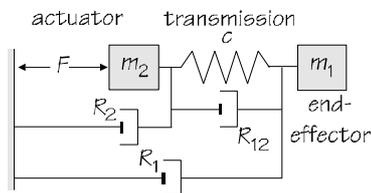


Figure 1: Typical mechatronic plant.

Often uncertainties are represented such that they lend themselves well for mathematical analysis. For robust controller design it is more appropriate to use a *physically motivated* uncertainty representation (Ackermann, 1993). To emphasize the importance of this insight, Ackermann postulates the ‘first basic rule of robust control’: *Require robustness of a control system only for physically motivated parameter values and not with respect to arbitrarily assumed uncertainties of the mathematical model.*

This paper addresses the problem of dealing with physical parameter uncertainty during controller design with a mechatronic design approach. First the application of Quantitative Feedback Theory (Horowitz, 1982) to this problem is motivated. Secondly several techniques are discussed that transform the description of structured uncertainty into a form suitable for Quantitative Feedback Theory (QFT). Next the techniques are evaluated for application to mechatronic systems. For one technique computer-support is developed to enhance the automated design of QFT-controllers.

2 QUANTITATIVE FEEDBACK THEORY

QFT is a design method for robust controllers that is able to deal with parametric uncertainty in a non-conservative way, while minimizing the cost of feedback (Horowitz, 1982).

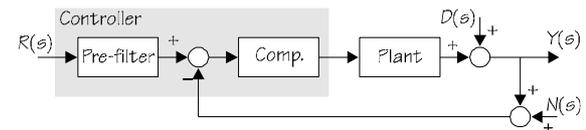


Figure 2: Feedback system (2-DOF).

Figure 2 shows the configuration of a QFT-controller that consists of a compensator and a pre-filter which will be designed consecutively.

The specifications for the controlled system have to be formulated as a desired frequency response band in a Bode diagram, e.g. as shown by the lighter

shaded area in Figure 3. The bounds **a** and **b** determine the performance specification. The freedom between the bounds gives the robustness specification. For all possible uncertainties the actual frequency responses of the controlled system (B) has to lie within this specified frequency band.

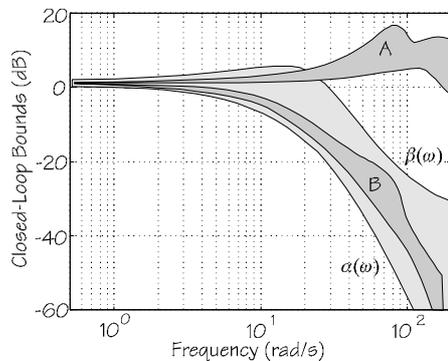


Figure 3: Frequency response bands.

The compensator is designed such that in spite of disturbances the robustness specification is met. I.e., the width of the frequency band A in Figure 3, that indicates the response of the loop for all combinations of uncertain physical parameters, has to be smaller than the width of the desired frequency response band. The pre-filter is designed such that the performance specification is met, i.e. the frequency band of the command response B is fitted into the desired response frequency band.

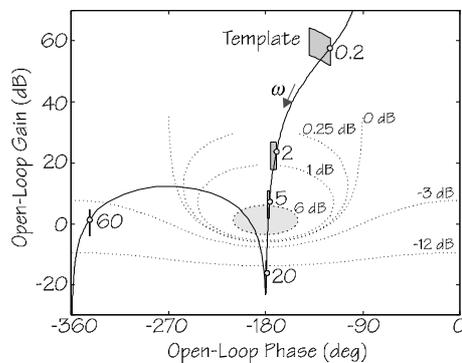


Figure 4: Templates in the Nichols chart.

Uncertainty is only dealt with for a selected set of frequencies, the so-called *critical frequencies*. For each critical frequency the structured uncertainty of the plant has to be specified as an uncertainty region in the Nichols chart, i.e. a *template* (shaded area in Figure 4). Evaluating the loop transfer function of an uncertain system for a particular frequency will result in a template in the Nichols chart, instead of

one point. The contour of this template is used for loop-shaping in the Nichols chart (Horowitz, 1982).

This paper addresses the problem of constructing templates for systems with uncertain physical parameters. It is important to determine the contour of a template exactly. Over-bounding requires extra efforts to make the system robust for non-existing uncertainties. This often results in a higher bandwidth and thus a higher cost of feedback, which is undesired. The advantage of QFT to handle structured uncertainty non-conservatively is lost.

3 TEMPLATES AND VALUE SETS

The physical parameters in a plant model are gathered in a physical parameter vector **q**. The elements of **q** are assumed to lie between mutually independent bounds.

$$\mathbf{q} = [q_1 \quad q_2 \quad \dots \quad q_k]^T \quad q_i \in [q_{i,-}, q_{i,+}] \quad (1)$$

In case the lower bound does not equal the upper bound, q_i is said to be *uncertain*. In the parameter space the set of possible uncertain parameter vectors is given by a hyperrectangle, the so-called *Q-box*.

$$Q = \{ \mathbf{q}_u = [q_1 q_2 \dots q_l]^T \mid q_i \in [q_{i,-}, q_{i,+}], i = 1, 2, \dots, l \} \quad (2)$$

The plant to be controlled $P(s, \mathbf{q})$ is a member of the parametric plant family $P(s, \mathbf{q})$. The *Q-box* has to be mapped onto templates. Thus the problem of finding the template contour is replaced by finding parameter values in *Q* that will be mapped onto the contour. Both the numerator and the denominator of the transfer function of $P(s, \mathbf{q})$ are polynomials in s with uncertain coefficients a_i :

$$p(s, \mathbf{q}) = a_0 + a_1 s + \dots + a_n s^n \quad (3)$$

The complex number taken by a polynomial when evaluated at $s = j\omega$ is called its *value*. Computing this value for each \mathbf{q}_u in *Q* yields the *value set*

$$p(j\omega, Q) = \{ p(j\omega, \mathbf{q}) \in C \mid \mathbf{q}_u \in Q \} \quad (4)$$

where $p(j\omega, \mathbf{q}_u)$ is a point in the complex plane *C*. A value set is a region in the complex plane that is fully determined by its contour. It can easily be transformed into a sub-template: the element $(\text{Re}(p), \text{Im}(p))$ in the value set corresponds to the point

$$\left(\arctan \left(\frac{\text{Im}(p)}{\text{Re}(p)} \right), 20 \log \left(\sqrt{\text{Re}^2(p) + \text{Im}^2(p)} \right) \right) \quad (5)$$

in the Nichols chart. The contour of a value set is mapped onto the contour of the corresponding template in the Nichols chart. There are two classes of methods to obtain the contour of a value set (Ackermann, 1993) and hence of a template:

A priori: determine *in advance* which physical parameter combinations (possibly) yield points at the contour of the template and next calculate the corresponding phase and magnitude.

A posteriori: calculate the frequency response for all relevant parameter combinations and determine *afterwards* which phase-magnitude combinations indeed contribute to the contour of the template.

4 A PRIORI TEMPLATE DETERMINATION

An uncertain plant family can be written as the division of two uncertain polynomial families

$$P_p(j\omega_i, \mathbf{q}, Q) = \frac{Num(j\omega_i, \mathbf{q}, Q_N)}{Den(j\omega_i, \mathbf{q}, Q_D)} \quad (6)$$

A template $R_p(\omega_i)$ can be determined a-priori if the numerator and the denominator of the transfer function of the plant depend on different parameters (Ackermann, 1993). In that case, the uncertain physical parameter vector \mathbf{q}_u can be split in $\mathbf{q}_{u,N} \in Q_N$ for the numerator and $\mathbf{q}_{u,D} \in Q_D$ for the denominator. Figure 5 shows the procedure for a priori template determination. Each step will be discussed hereafter.

4.1 Template addition in Nichols chart

The uncertain numerator and denominator families

$$\begin{aligned} Num(j\omega_i, \mathbf{q}, Q_N) &= \{num(j\omega_i, \mathbf{q}_{u,N}) \mid \mathbf{q}_{u,N} \in Q_N\} \\ Den(j\omega_i, \mathbf{q}, Q_D) &= \{den(j\omega_i, \mathbf{q}_{u,D}) \mid \mathbf{q}_{u,D} \in Q_D\} \end{aligned} \quad (7)$$

can both be represented in the Nichols chart by sub-templates R_N and R_D . Template R_p can be

constructed by subtracting any denominator vector from any numerator vector (Figure 6), i.e. the last step in Figure 5:

$$\arg(P(j\omega_i, \mathbf{q}_u)) = \arg(num(j\omega_i, \mathbf{q}_{u,N})) - \arg(den(j\omega_i, \mathbf{q}_{u,D})) \quad (8)$$

$$\begin{aligned} |P(j\omega_i, \mathbf{q}_u)|_{dB} = \\ |num(j\omega_i, \mathbf{q}_{u,N})|_{dB} - |den(j\omega_i, \mathbf{q}_{u,D})|_{dB} \end{aligned} \quad (9)$$

Let n_0 be the nominal numerator in sub-template R_N and let d_0 be the nominal denominator in sub-template R_D (Figure 6a). If the denominator would be certain, mirrored sub-template $R_{D,mirror}$ would degenerate to nominal mirrored denominator $-d_0$. Template R_p would equal the shifted sub-template $R_N^* = R_N - d_0$, and nominal plant P_0 would correspond to the shifted nominal numerator $n_0 - d_0$ (Figure 6b). In case the denominator is *uncertain*, the uncertainty in the entire plant increases. The shifted sub-template R_N^* must be extended with those points that can be reached within mirrored sub-template $R_{D,mirror}$, when the nominal point $-d_0$ lies within sub-template R_N^* . R_p thus can be constructed by shifting the mirrored sub-template $R_{D,mirror}$ around sub-template R_N^* , keeping the nominal point of sub-template $R_{D,mirror}$ on the contour of R_N^* (Figure 6c). Template R_p (Figure 6d) can be regarded the result of the addition of the sub-templates R_N and $R_{D,mirror}$.

Next the problem of constructing sub-templates for uncertain polynomials, the first two steps in Figure 7, will be addressed. (Transformation of a value set into a template is performed by (5)). Depending on the type of polynomial the contour of a value set can be calculated (Ackermann, 1993). For the determination of a value set, it is important that the polynomial is classified in the simplest category.

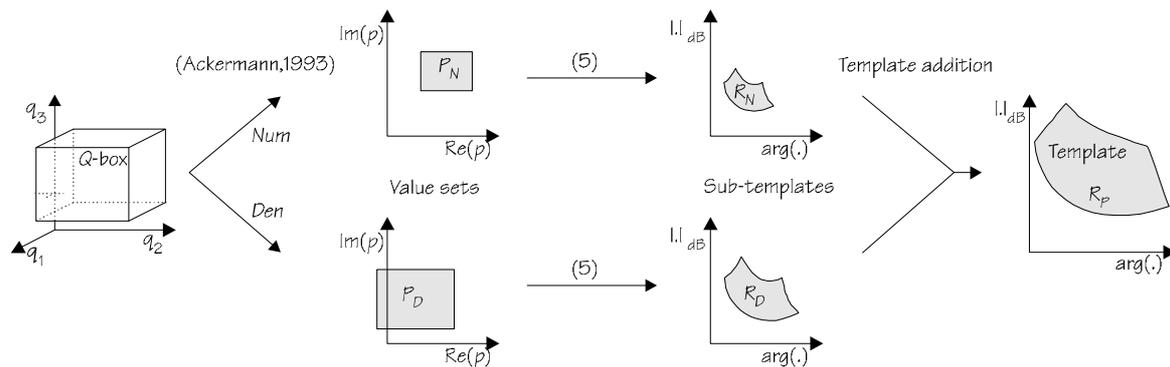


Figure 5: A priori template determination.

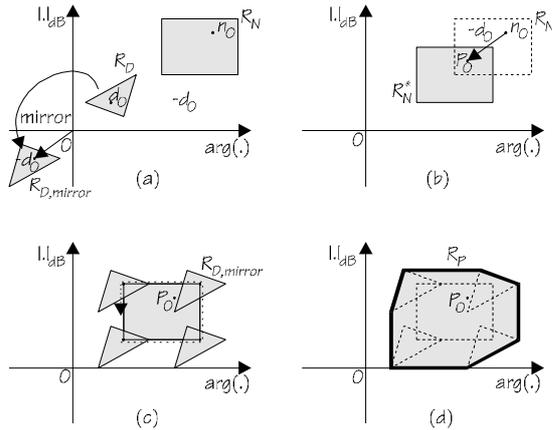


Figure 6: Template addition.

4.2 Interval coefficients

An *interval polynomial* is a polynomial for which the uncertain coefficient vector \mathbf{a} ranges over the box

$$A = \{ \mathbf{a} \mid a_i \in [a_{i,-}; a_{i,+}], i = 0, 1, \dots, n \} \quad (10)$$

For an interval polynomial a_i is not considered a function of other parameters. The contour of a value set is the rectangle between the complex points corresponding to the Kharitonov polynomials:

$$\begin{aligned} p_{+-}(s) &= a_{0,+} + a_{1,-}s + a_{2,-}s^2 + a_{3,+}s^3 + a_{4,+}s^4 \dots \\ p_{++}(s) &= a_{0,+} + a_{1,+}s + a_{2,-}s^2 + a_{3,-}s^3 + a_{4,+}s^4 \dots \\ p_{-+}(s) &= a_{0,-} + a_{1,+}s + a_{2,+}s^2 + a_{3,-}s^3 + a_{4,-}s^4 \dots \\ p_{--}(s) &= a_{0,-} + a_{1,-}s + a_{2,+}s^2 + a_{3,+}s^3 + a_{4,-}s^4 \dots \end{aligned} \quad (11)$$

This rectangle can be transformed to a template in the Nichols chart using (5), refer Figure 5.

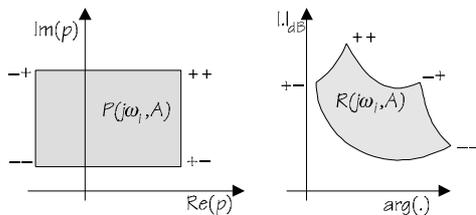


Figure 7: Value set and template for interval polynomial.

4.3 Affine coefficients

For affine polynomials all coefficients are affine functions of the uncertain parameter vector \mathbf{q}_u .

$$a_i(\mathbf{q}_u) = c_i + \mathbf{d}_i^T \mathbf{q}_u \quad (12)$$

The Q -box is mapped onto a closed convex polygon $R(\omega, \mathbf{q}_u)$. All edges of the polygon R correspond to edges of Q . The contour of the value set is obtained

by calculating the polynomial value for all vertices of the parameter space Q and next determining the convex hull of these points in the complex plane.

Consider the following polynomial with uncertain parameters $q_1, q_2, q_3 \in [-1, 1]$.

$$p = q_1 + 2q_2 + (q_2 + 2q_3)s + (q_3 + 2q_1)s^2 + s^3 \quad (13)$$

The value set for $\omega=1$ is given in Figure 8. The points marked with +++, -++ etc. correspond to vertices of Q . The edges of Q are mapped onto the straight lines between these points. The contour of the value set is the convex hull of these straight lines.

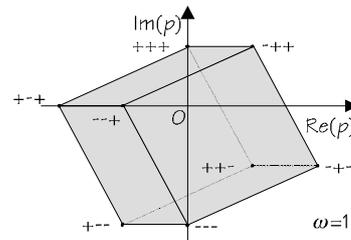


Figure 8: Template for affine polynomial.

4.4 Multilinear coefficients

Multilinear coefficient functions are functions with terms like $q_1q_2, q_2q_3, q_1q_2q_3$ etc, but no terms like q_1^2 or $q_1q_2^2$. For multilinear polynomials, the contour of the value set may correspond to extreme values of one of the physical parameters, but interior points of Q may also contribute to the contour. An illustration of this fact is given in Figure 9. The edges of Q are mapped onto the four straight lines A,B,C,D, creating the lighter shaded area. However, an interior line in the Q -box is mapped onto the curve *outside* this polygon, thus creating the darker shaded area.

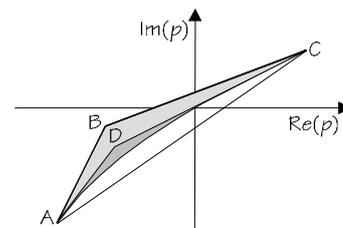


Figure 9: Template for multilinear polynomial.

For multilinear polynomial families a rough approximation of the value set may be obtained with the use of the mapping theorem of Desoer (Ackermann, 1993): *The convex hull of the value set of a polynomial with multilinear coefficient functions is the convex hull of the images of the vertices of Q .* Thus creating the polygon ABC in Figure 9, which is somewhat conservative.

4.5 Dependency numerator and denominator

For physical plants, the numerator and the denominator of a transfer function typically share dependency on some uncertain physical parameters. The a priori template contour determination method presented here will yield an over-bounded template. This is illustrated for the transfer function from input force to actuator position for the system of Figure 1:

$$P(s, \mathbf{q}) = \frac{b_1 s + b_0}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s}$$

$$\begin{aligned} b_1 &= R_{12} \\ b_0 &= c \\ a_4 &= m_1 m_2 \\ a_3 &= (R_1 m_2 + R_{12} m_1 + R_{12} m_2 + R_2 m_1) \\ a_2 &= (c m_2 + c m_1 + R_1 R_2 + R_{12} R_1 + R_{12} R_2) \\ a_1 &= c(R_1 + R_2) \end{aligned} \tag{14}$$

The mass of the end-effector m_1 is unknown but bounded; it may vary between 100% and 200% of its nominal value. The stiffness c may vary between 50% and 100% of its nominal value. Other physical parameters are assumed to be certain.

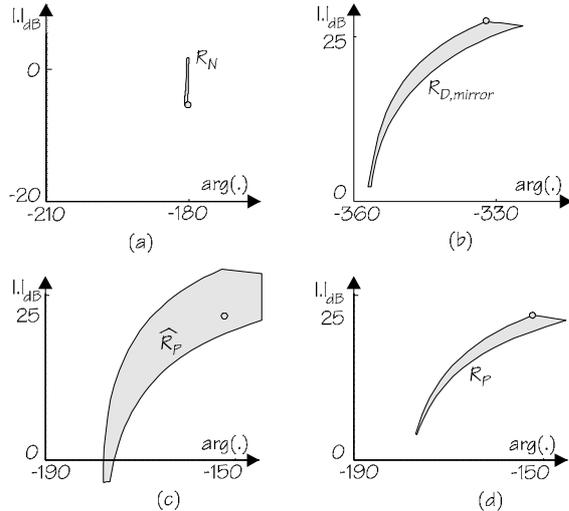


Figure 10: a) shifted numerator sub-template
 b) denominator sub-template
 c) overbounded template
 d) actual template

The uncertain parameters occur in both the numerator and the denominator. Figure 10a shows the sub-template R_N of the numerator. The mirrored sub-template $R_{D,mirror}$ of the denominator, with the nominal point set to the origin, is shown in Figure

10b. Addition of the templates results in the over-bounded template of Figure 10c. The actual template R_P is much smaller, as can be seen from Figure 10d. In this case application of a priori template determination (Figure 7) results in over-bounding.

5 A POSTERIORI TEMPLATE DETERMINATION

The contour of a template can also be determined a posteriori. The response $(\arg(P(j\omega, \mathbf{q})), |P(j\omega, \mathbf{q})|_{dB})$ can be calculated for a finite set of parameter vectors in the Q -box. This subset is referred to as the *grid* and the approximation technique is referred to as *gridding* (Ackermann, 1993). The problem in gridding is to determine the density of the grid beforehand. Visual inspection of the template however yields an immediate answer to the question whether an applied grid was dense enough or not. Therefore it is advisable to start with a relatively low grid density that can be iteratively increased (Figure 11 and Figure 12). Once the gridding result is dense enough one has to determine the points of the template that contribute to the contour.

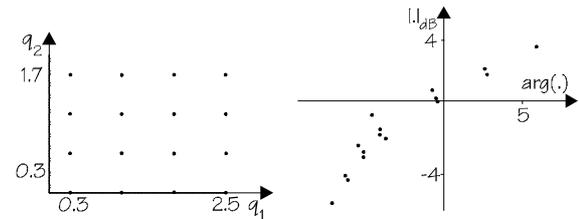


Figure 11: Gridded Q-box and gridding result.

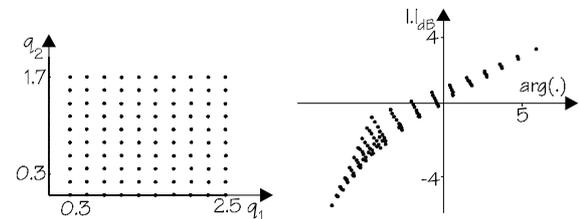


Figure 12: A denser grid, a better gridding result.

6 TEMPLATES FOR MECHATRONIC SYSTEMS

A priori determination of the contour of a template is only applicable for a small group of models. In case the uncertainty shows complexity it is difficult to determine beforehand which physical parameter combinations contribute to the template contour. For this reason many controller design problems do not take parametric uncertainty into account quantitatively. A typical mechatronic design problem would be to make the system of Figure 1 robust for

load variations and stiffness uncertainty. Even for the relatively simple transfer function (14) it is not straightforward to determine the template contours a priori, as the numerator and denominator are not mutually independent. The stiffness occurs in both polynomials.

Gridding is a straightforward, maybe time-consuming, but nearly errorless method that can easily be automated. The *a posteriori* method will be used to enhance the automated design of QFT-controllers for mechatronic systems with uncertain physical parameters. An interesting feature is that the designer can select grid-points on the contour of the template and investigate which combination of parameter values corresponds to that particular point. In this way, the mechatronic designer is given the opportunity to signal physical parameter uncertainties that unnecessarily complicate controller design. The designer could reconsider the plant design in order to avoid undesired situations.

7 COMPUTER-BASED SUPPORT

Computer-based support has been developed that automatically converts a physical plant model with uncertain physical parameters into templates. First the physical plant model is built in 20-sim (Weustink et al., 1998). Physical parameter values are entered as a number or as a range, as in Figure 13.

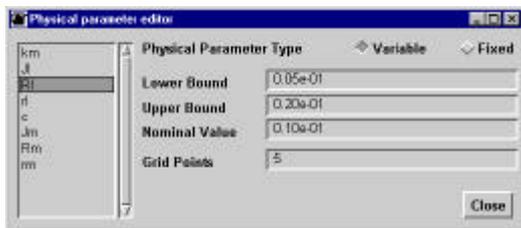


Figure 13: The physical parameter editor.

For uncertain parameters the number of grid points have to be specified. A symbolic plant model is exported to Matlab together with the data concerning the physical parameters. A special purpose M-file calculates the frequency responses for the specified grid, and draws the templates using the Matlab QFT toolbox (Borghesani et al., 1994). Figure 14 shows a template for a specific frequency.

The designer should indicate the contour of the template that is needed for the QFT design. The QFT-toolbox (Borghesani et al., 1994) can now be used for controller design.

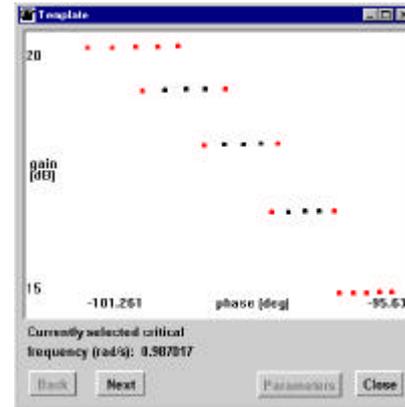


Figure 14: The template editor.

8 CONCLUSIONS

Controllers for mechatronic systems should be robust for physical parameter uncertainty without a high cost of feedback. With Quantitative Feedback Theory these controllers can be designed, but uncertain physical parameters have to be mapped onto templates in the Nichols chart. Methods for a priori template determination depend on the type of polynomials of the plant transfer function and can only be used when uncertain physical parameters do not occur in both the numerator and denominator. A posteriori template determination (gridding) is a straightforward and nearly errorless method that can be applied to all transfer functions. This technique is the most appropriate for mechatronic systems. Computer-based support has been developed that automatically grids the parameter space of a linear time-invariant plant model and that automatically determines the templates. These can be used by the Matlab QFT-toolbox. The designer is given the opportunity to signal physical parameter uncertainty that unnecessarily complicates controller design.

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