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AUTOMATED OPTIMAL CONTROLLER DESIGN IN MECHATRONICS

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Abstract: Optimal control requires a linear model of the system to be controlled and a cost criterion that indicates the objectives of the controlled system. Once these are available, the parameter values for the controller can be found procedurally using suitable software tools. This paper proposes a procedure to determine this criterion in the conceptual design stage, for a particular class of mechatronic systems. A rule of thumb is combined with a mechatronic design tool that can be automated almost completely. Therefore, the proposed design procedure of optimal control with guaranteed performance and stability robustness can also be automated completely.

Keywords: servo systems, control system design, PD controllers, optimal control, methodology, computer aided control system design

1. INTRODUCTION

A large group of electro-mechanical systems that contain an actuator and an end-effector can be represented competently by a fourth-order model (figure 1), as they contain one dominant mechanical compliance (Oelen, 1994; Koster *et al.*, 1995). For this class of systems, Groenhuis (1991) developed a design tool for the minimization of the positional error after a point to point motion. It is a powerful mechatronic design tool in the *conceptual design* stage, as it provides the necessary interaction between the design of the controller, the electro-mechanical system and the determination of the input function. It ensures an *upper bound* for the positional error and guarantees a sufficient stability margin. It is based on the use of dimensionless quantities, which provides the opportunity to draw general conclusions for this class of systems, regardless of the particular parameter values.
The Groenhuis method is attractive for mechatronic controller design, because little information about the plant is needed, while a reliable worst case performance prediction is obtained.

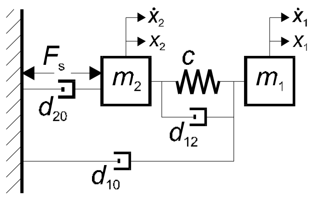

Fig. 1. System with dominant compliance, where m_1 is generally referred to as the load and m_2 as the motor.

For optimal control, nothing similar is available. The goal of this paper is to establish a procedure that provides a reliable worst case performance prediction for the same class of systems with state-feedback, using the same initial knowledge.
Section 2 gives an outline of the Groenhuis design tool. Section 3 shortly describes the optimal control problem. In section 4 the choice of weighting matrices is discussed, including a design procedure for an optimal controller for the class of fourth-order systems. In section 5 results are evaluated and section 6 presents the conclusions.

2. GROENHUIS DESIGN TOOL

System description
We consider a system consisting of a *plant*, a *controller* and a *reference path generator*, configured as shown in the scheme of figure 2.

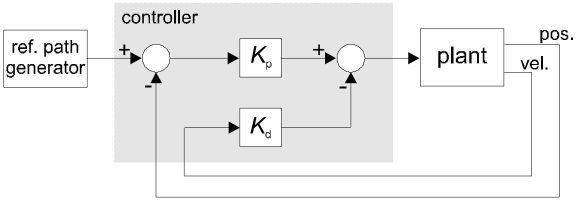

Fig. 2. Controlled system configuration.

The plant can be described as in figure 1. The controller implements a proportional action K_p on a positional error and a proportional action K_d on one measured velocity. The positional error is obtained as the difference between one measured position and the reference path. The reference path is a smooth function. The smoothness is determined by an order number. If the reference path is of order 2, it involves two pieces of 2nd order polynomials (*i.e.*, it is a B-spline of order 3). If it is of order 3, it involves two pieces of 3rd order polynomials.

Determination of dimensionless plant properties
Mass ratio:

$$a = \frac{m_2}{m_1 + m_2} \quad (1)$$

Resonance frequency when motor position is fixed:

$$w_m = \sqrt{\frac{c}{m_1}} \quad (2)$$

Problem specification
The reference path specifies, in terms of the end effector m_1, a movement over a distance h_m in a time period t_m. The control goal is to guarantee an upper bound e_0 on the absolute value of the positional error after the reference path has reached the endpoint.

Determination of dimensionless problem spec
Maximal relative positional error:

$$E_0 = \frac{e_0}{h_m} \quad (3)$$

Dimensionless problem–plant relations
Crucial is the periodic ratio t_m. This ratio can be calculated in two ways:

– from the characteristic plant eigen period and the movement period:

$$t_m = \frac{T_m}{t_m} = \frac{2p}{w_m t_m} \quad (4)$$

– from the maximal relative positional error and path order, see table 1.

Table 1 Periodic ratio per path order

Order	Periodic ratio	Rel. error	
2	$t_m = \sqrt{5E_0}$	$E_0 = 0.2 \cdot t_m^2$	(5a)
3	$t_m = \sqrt[3]{1.66E_0}$	$E_0 = 0.6 \cdot t_m^3$	(5b)

Relations (4) and (5) can be used in various ways, depending upon the specific design context. If plant characteristics and motion duration and distance have been chosen, (4) can be used to determine t_m, and (5) can be used to obtain the reference path order and corresponding e_0. If on the other hand e_0 and the motion distance are known, (5) can be used to determine t_m, and (4) may help to evaluate the required stiffness c for a particular motion duration, etc.

Choice of sensor location
Four different configurations for sensor locations are possible. As we are interested primarily in load position, it is not sensible to feed back load velocity and motor position. Hence, three options remain (original labeling of Groenhuis has been maintained), see table 2.

Table 2 Possible sensor locations

Label	I	III	VI
Feedback	$x_2, \dot{x}_2$	$x_1, \dot{x}_2$	$x_1, \dot{x}_1$
Condition	$0.1<a<0.8$	$0.1<a<0.5$	$0.1<a<0.12$

A choice of sensor position leads to optimal dimensionless controller settings, as given in table 3.

Table 3 Recommended dimensionless controller settings (positional error).

Label	I	III	VI
Ω_p	0.8	0.6	1.0
Ω_d	1.0	0.6	0.4

Recommended controller settings.
The values for the proportional control actions, for a particular problem setting, are:

$$K_{\mathrm{p}} = \frac{\Omega_{\mathrm{p}}^{2} \cdot c}{1 - a} \qquad (6)$$

$$K_{\mathrm{d}} = \frac{\Omega_{\mathrm{d}} \cdot \sqrt{m_1 c}}{1 - a} \qquad (7)$$

Method of research.
The optimal dimensionless controller settings from table 3 are obtained by performing numerous simulations for a relevant set of these dimensionless settings. The setting that results in the smallest positional error, after the reference path has reached the end-point, and that provides a sufficient stability margin is the optimal setting.

Stability robustness.
To provide stability robustness a stability margin was introduced, that defines a minimal negative value d for the real part of the poles of the controlled system. The dimensionless version of this margin is:

$$D = \frac{d}{w_{\mathrm{m}}} \qquad (8)$$

Sufficient stability margin was attained for $D = 0.2$.

Tracking systems
The method of research Groenhuis applied has been repeated for the same class of systems, but with the objective to minimize the tracking error. Two configurations are considered:
– measurement of the motor position (concept I);
– measurement of the load position (concept VI).
The velocity is reconstructed from the positional error using a tame D action in the controller. The optimal dimensionless quantities are given in table 4:

Table 4 Recommended dimensionless controller settings (tracking error).

Label	I	VI
Ω_{p}	0.5	1.1
Ω_{d}	0.7	1.1

Equations (6) and (7) can be reused to obtain the proportional and derivative gain for a particular system. Also (4) can be reused. For a second-order input function (5) has to be replaced by the following relation between E_{track} and the ratio t_{m}:

$$E_{track} = 0.7 \cdot t_{\mathrm{m}}^{2} \qquad (9)$$

3. OPTIMAL CONTROL

Optimization is used to maximize or minimize objectives which are generally expressed in a criterion. In case of optimal control the *linear quadratic regulator* (LQR) problem for time-invariant plants can be formulated as:

Given the linear time-invariant system

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{Ax} + \boldsymbol{Bu} \\ \boldsymbol{y} &= \boldsymbol{Cx} \end{aligned} \qquad (10)$$

find a control function $\boldsymbol{u}(t)$ that will minimize the cost criterion J given by:

$$J = \frac{1}{2}\int_0^\infty (\boldsymbol{x}'\boldsymbol{Qx} + \boldsymbol{u}'\boldsymbol{Ru})\, dt \qquad (11)$$

with the weighting matrices $\boldsymbol{Q}$ and $\boldsymbol{R}$ usually symmetric, $\boldsymbol{R}$ positive definite and $\boldsymbol{Q}$ positive semi-definite (Stefani *et al.*, 1994).

Once the process and the cost criterion are known the state-feedback controller $\boldsymbol{K}$ can be determined by finding the unique positive semi-definite solution $\boldsymbol{P}$ of the algebraic Riccati equation.

$$\boldsymbol{K} = -\boldsymbol{R}^{-1}\boldsymbol{B}^{T}\boldsymbol{P} \qquad (12)$$

The problem is not to determine the state-feedback controller $\boldsymbol{K}$, as a software tool like Matlab can perform this task automatically. Rather, the problem is to express the control objectives in terms of a proper cost criterion J.
To provide the possibility to tune the controller, the relation between the cost criterion, the plant properties and the control objectives should be transparent. Thereby it should be noted that only the relative size of the weighting matrices $\boldsymbol{Q}$ and $\boldsymbol{R}$ influence the controller settings. They indicate a trade-off between control performance and low-energy input. In case $\boldsymbol{Q}$ is larger than $\boldsymbol{R}$, the states unequal to zero are minimized relatively more than the control signals and *vice versa*. Elements in the weighting matrix $\boldsymbol{Q}$ determine the relative minimization in between states (diagonal elements) or combinations of states (off-diagonal elements).

4. CHOICE OF WEIGHTING MATRICES

The actual values of the weighting matrices $\boldsymbol{Q}$ and $\boldsymbol{R}$ may be determined by inverse optimal control in case the feedback vector is known. The feedback vector of the PD-controller has two entries that equal zero, therefore it is not optimal (Kalman, 1964). Another possibility is to use *a rule of thumb* (Bosgra and

Kwakernaak, 1995), that starts from diagonal weighting matrices $\boldsymbol{Q}$ and $\boldsymbol{R}$:

$$\boldsymbol{Q} = \begin{bmatrix} Q_1 & 0 & .. & 0 \\ 0 & Q_2 & .. & 0 \\ .. & .. & .. & .. \\ 0 & 0 & .. & Q_n \end{bmatrix} \tag{13}$$

$$\boldsymbol{R} = \begin{bmatrix} R_1 & 0 & .. & 0 \\ 0 & R_2 & .. & 0 \\ .. & .. & .. & .. \\ 0 & 0 & .. & R_k \end{bmatrix} \tag{14}$$

where

$$\sqrt{Q_i} = \frac{1}{z_i^{\max}}, \; i = 1,2,\ldots,n. \tag{15}$$

$$\sqrt{R_i} = \frac{1}{u_i^{\max}}, \; i = 1,2,\ldots,k. \tag{16}$$

The number $z_i^{\max}$ denotes the maximally acceptable deviation value for the *i*-th component of the output *z*. The other quantity $u_i^{\max}$ has a similar meaning for the *i*-th component of the input *u*.
It is not easy to determine these quantities beforehand. The following design procedure will give a good initial setting:

1. Design a Groenhuis PD-controller.
2. Perform simulations with the controlled system. As an optimal controller is basically designed for regulator systems, the simulations with the PD-controller will be performed with the same purpose. Two states, *i.e.* positions of m_1 and m_2, will get an off-set at start-up and the PD-controller will steer the process back to its equilibrium.
3. The first overshoot of the response of the control signal and the errors of the motor and load position determine the values of the three variables, using (15) and (16).
4. Calculate weighting matrices $\boldsymbol{Q}$ and $\boldsymbol{R}$ with equations (13) and (14).
5. Determine the LQR-controller using e.g. Matlab.

By means of simulation experiments it has been verified that using only position related entries in $\boldsymbol{Q}$ and $\boldsymbol{R}$ results in a better performance than using only the position of the load or taking into account the velocities. Tuning of the initial setting of the weighting matrices is still possible, as the non-zero elements in the matrices have a simple physical interpretation.
For all configurations from table 2 and the complete range of the dimensionless parameters this procedure has been tested, such that general conclusions can be drawn for the class of systems that can be represented by figure 1. The mass-ratios that have been considered for the different configurations are shown in table 5.

Table 5 Mass-ratios per feedback configuration.

Config.	I	III	VI
a = 0.1	✓	✓	✓
a = 0.5	✓	✓	.
a = 0.8	✓	.	.

The design procedure for the optimal controller has been practically verified on the Linix laboratory setup (figure 3), that has been especially designed to represent the considered class of mechatronic systems. It contains optical position encoders on both axes and angular velocities are obtained using state variable filters. The experiment uses concept I and the following parameter values: c = 1.98 [Nm rad^{-1}], a = 0.21, w_m = 27.5 [rad sec^{-1}] and t_m = 2 [sec].

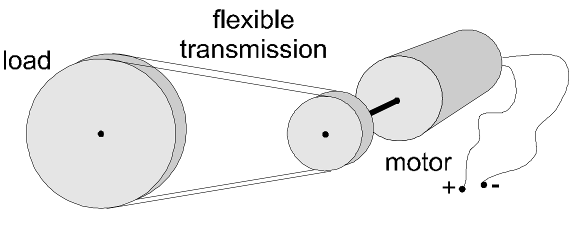

Fig. 3. The Linix laboratory setup.

1. The parameters for the Groenhuis PD-controller configuration I are:

$$\begin{aligned} K_p &= 1.61 \quad [\text{Nm}] \\ K_d &= 0.092 \quad [\text{Nm sec}^{-1}] \end{aligned} \tag{17}$$

2. An initial offset with amplitude equal to 1 is applied to both positions, *i.e.* the angles of the pulleys. The simulations, performed in 20-sim (Controllab Products inc.), are shown in figure 4.

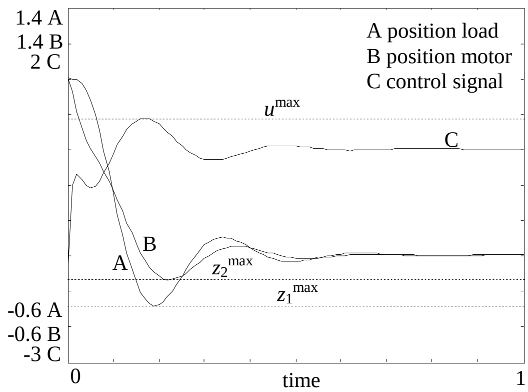

Fig. 4. Simulations of PD-controller as regulator.

3. From figure 4 the maximal deviations can be found. Note that the deviations are maximal at t=0, but these are due to the initial settings and do not give any information about the behavior of the system. Therefore, the next maximum (or minimum) is used. The maximal deviations are:

$$\begin{aligned} z_1^{\max} &= -0.2848 \\ z_2^{\max} &= -0.1391 \\ u^{\max} &= 0.4365 \end{aligned} \qquad (18)$$

4. With these deviations the matrices $\boldsymbol{Q}$ and R are derived using (13)-(16). Note that the states are labeled according figure 1, the position of the load respectively the motor. The states x_3 and x_4 equal the derivatives of x_1 and x_2, *i.e.* the velocities of the load respectively the motor.

$$\boldsymbol{Q} = \begin{bmatrix} 12.33 & 0 & 0 & 0 \\ 0 & 51.67 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R = 5.25 \qquad (19)$$

5. Standard Matlab functions are used to determine the feedback vector and the DC gain of the optimal controlled system:

$$\begin{aligned} \boldsymbol{K} &= \begin{bmatrix} 3.1845 & 2.0092 & 0.2649 & 0.0526 \end{bmatrix} \\ DC\ gain &= 0.1162 \end{aligned} \qquad (20)$$

The optimal controller is now compared to the Groenhuis PD-controller on the same system, using a second-order input function, with an amplitude of $2p$ rad. The simulation result for the PD-controlled system is:

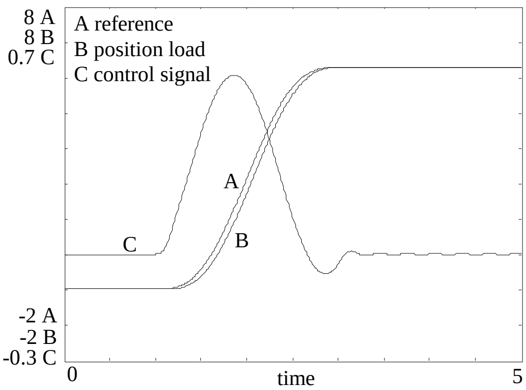

Fig. 5. Simulated response of PD-controlled system.

The reference signal of the LQR-controlled system is divided by the DC-gain, to adapt the regulator for tracking purposes. The simulation result of this system with same input function as above is shown in figure 6.

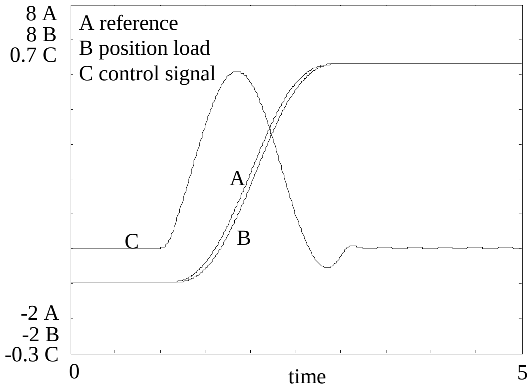

Fig. 6. Simulated response of LQR controlled system.

Practical experiments have been performed to verify the simulation results. In figure 7 the relevant parts of the responses of the Linix with both PD and LQR controller are shown. They confirm the simulation results, as far as the dynamical part of the response is concerned. However, both situations suffer from a steady-state error due to stiction on both axes.

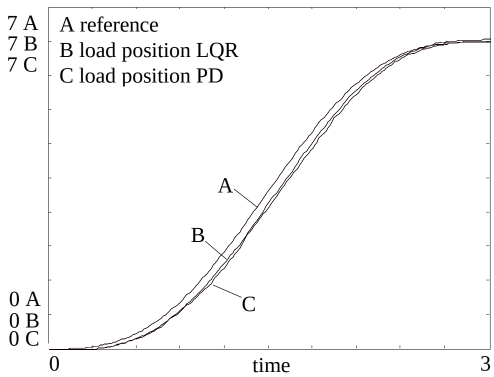

Fig. 7. Two practical responses of the Linix.

5. EVALUATION

Comparison of the simulations (figure 5 and 6) shows that both the tracking error and the positional error are smaller when using the LQR controller. The experiment also shows a smaller tracking error for the LQR controlled systems (figure 7). Note that the objective is not to design a better controller, but a controller with the same guaranteed performance and stability margin. To get an impression of the accuracy that can be obtained with the LQR controller, the relation between the errors and the periodic ratio t_{m} is determined using Matlab. For the relative positional error this relation is shown in figure 8 and for the tracking error this is shown in figure 9, for all situations summarized in table 5. The two upper bounds of the LQR controlled system equal:

$$E_0 = 0.12 \cdot t_{\mathrm{m}}^2 \qquad (21)$$

$$E_{\mathrm{track}} = 0.5 \cdot t_{\mathrm{m}} \qquad (22)$$

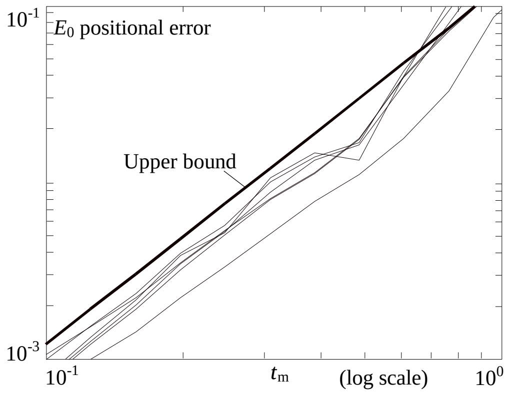

Fig. 8. Positional error E_0 as function of t_m.

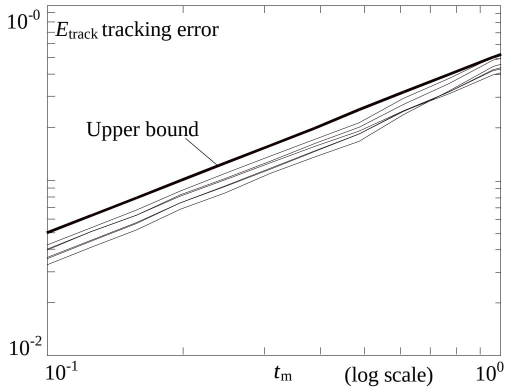

Fig. 9. Tracking error E_{track} as function of t_m.

These relations (21-22) are only valid for the initial settings, obtained according (13)-(16). They form an upper bound for the errors, for all feedback configurations and mass ratios that are considered. Comparison of these expressions with similar expressions for the Groenhuis PD-controller (5) and (9), show a smaller error for all values of t_m. So an expression for an improved guaranteed performance prediction is available.

The practical response of the optimal controlled system shows a tracking error that is within the margins described by (21). The positional error is larger in both cases due to the stiction on both axis.

It appeared that the resulting optimal controllers derived from the PD controlled system according to configuration I and configuration III are almost identical. This result offers the possibility to choose the sensor location in a later stage of the mechatronic design process. The Groenhuis design tool requires this choice to take place at an early stage. The advantage of a choice of sensor locations in a later stage of the design process is that in case a revision of the design or design specifications require different sensor positions, this is possible without major changes to the design and without repeating large parts of the design process.

The LQR controller requires all states to be available for measurements, which is an unrealistic constraint in practical situations. But an observer can be build, that uses the available sensor outputs of the plant and the control signal to reconstruct the states, according a second cost criterion. Which output is to be measured (*i.e.* choice of sensor position) is only determined at this stage of the design process. It may be the position of the load or the position of the motor, depending on the constraints the designer poses on the plant and the plant properties.

The combination of the LQR controller and an optimal observer is called an *LQG controller*. The LQG has no guaranteed stability margins, but has a better noise suppression. *Loop Transfer Recovery* (LTR) may be used to recover the properties of the LQR controller while maintaining the LQG machinery (Stefani *et al.*, 1994).

6. CONCLUSIONS

Groenhuis (1991) offers a design tool for a class of electro-mechanical systems using a mechatronic design approach. Most parts of the design procedure may be automated, except for the choice of sensor locations. An optimal controller may give a better performance and stability robustness, but the definition of the cost criterion is a problem. Simulations with the Groenhuis PD-controlled systems and a rule of thumb can be used to define a cost criterion, that leads to an LQR controller with a guaranteed stability margin and performance prediction for the class of fourth-order systems. This design procedure may be automated completely, while the choice of sensor locations is extended to a later stage.

REFERENCES

Bosgra, O.H. and H. Kwakernaak (1995), *Design Methods for Control Systems*, Notes for a course of the Dutch Institute of Systems and Control, The Netherlands.

Controllab Products inc., *20 sim Reference Manual*, Enschede, The Netherlands. WWW address: http://www.rt.el.utwente.nl/20sim/clp.htm

Groenhuis, H. (1991), *A Design Tool for Electromechanical Servo Systems*, PhD thesis, University of Twente, Enschede, The Netherlands.

Kalman, R.E., (1964), When Is a Linear Control System Optimal?, *Journal of Basic Engineering* **86**, pp. 51-59.

Koster, M.P., W.T.C. van Luenen and T.J.A. de Vries, *Mechatronica* (Mechatronics), in Dutch, course material, University of Twente, Enschede, The Netherlands.

Oelen, W. (1995), *Modelling as a tool for design of mechatronic systems*, PhD thesis, University of Twente, Enschede, The Netherlands.

Stefani, R.T., C.J. Savant, B. Shahian and G.H Hostetter (1994), *Design of Feedback Control Systems*, Saunders College Publishing, U.S.A.