MRAS : Model Reference Adaptive Systems

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1. INTRODUCTION

In recent years one of the major topics of research is that of adaptive control systems. A sufficiently general definition of adaptive control is difficult to give, because there are many structures which may be called adaptive in some sense. For our purpose the following definition is suitable :

"An adaptive system is a system where in addition to the basic (feedback) structure, explicit measures are taken to compensate for variations in the process dynamics or for variations in the disturbances, in order to maintain an optimal performance of the system."

The additional measures mentioned in this definition may either be adjustment of the controller parameters or generation of additional input signals. This leads to parameter adaptive systems and signal adaptive systems (Fig. 1a and 1b).

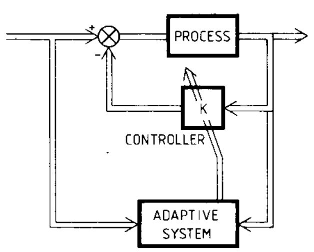

Fig. 1a. Parameter adaptive system

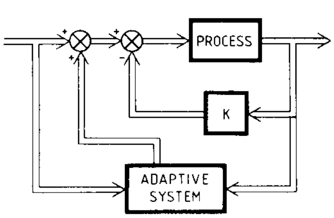

Fig. 1b. Signal adaptive system

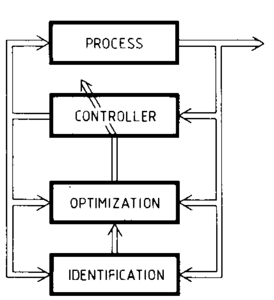

Fig. 2. Indirect adaptive system

Another distinction which can be made is between direct and indirect adaptive systems. In indirect adaptive systems the adjustment of the basic control system is based on on-line identification of the process, followed by optimization of the controller parameters, while in direct adaptive systems no explicit identification can be recognized. The basic structure of an indirect adaptive system is given in fig. 2.

A direct adaptive system is, for example, a system with the structure of figure 1a where the parameter adjustment is based on on-line minimization of a criterion with the aid of hill climbing techniques.

Model Reference Adaptive Systems (MRAS) basically also belong to the class of direct adaptive systems. Because MRAS can also be applied for identification it is possible to use MRAS in an indirect adaptive system as well.

Let us start by considering direct adaptive systems. In this case all structures have in common that the desired (optimal) performance of the system is defined by a reference model. This reference model, which has as input the system's reference signal, generates an output signal with the desired shape of response. The "conventional" control loop is adjusted by an adaptation mechanism which attempts to make the responses of process and reference model identical.

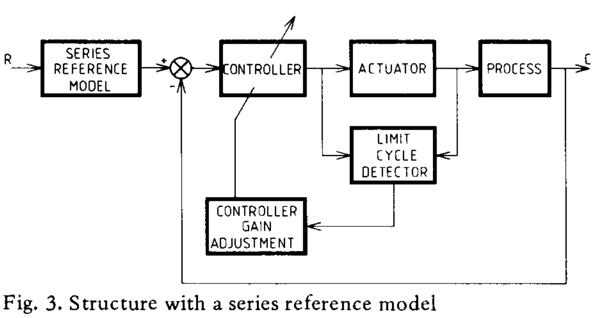

Fig. 3. Structure with a series reference model

In fig. 3 one of the earliest model reference adaptive systems is depicted. This system, developed by Whitaker et al. [15] was used for aircraft control. A series reference model generates the desired response. In order to realize a similar response of the process, the loop gain of the feedback control system

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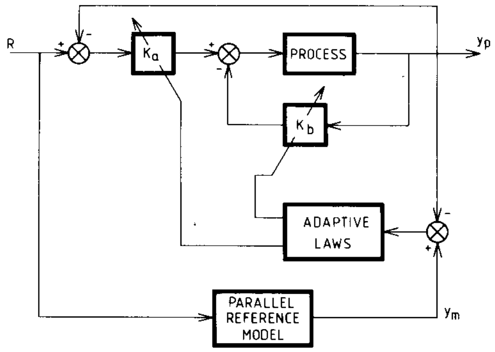

Fig. 4. Structure with a parallel reference model

should be as high as possible. This gain is automatically increased, until a limit cycle detector detects that the system is on the border of instability.
A presently commonly applied structure uses a parallel reference model (fig. 4). In this structure either the difference between the process output and the reference model output, the signal e, or the difference between the states of the process and of the reference model, the vector $\underline{e}$, is minimized.

2. DESIGN METHODS

2.1. The sensitivity concept

There are several approaches to designing the adaptive controller. The earliest designs use the sensitivity concept. The adaptive laws are derived as follows. Subtraction of the output signal of the process, yp, from the output signal of the reference model, ym, yields the error signal e, which is used to define a quadratic criterion C :

$$C = \frac{1}{2} \int_0^T e^2 \, dt \qquad (1)$$

Suppose that variations in the process dynamics can be compensated by adjusting the parameter Ki. In order to minimize C the parameter Ki has thus to be adjusted. The variations of Ki are chosen to be proportional with the gradient of C with respect to Ki.

$$\Delta Ki = -\alpha \frac{\partial C}{\partial Ki} \qquad (2)$$

A continuous adjustment law is found by differentiating eqn. (2) with respect to time. This yields

$$\frac{dKi}{dt} = -\alpha \frac{d}{dt} \frac{\partial C}{\partial Ki} \qquad (3)$$

Substituting eqn. (1) into eqn. (3) yields

$$\frac{dKi}{dt} = -\alpha \frac{\partial}{\partial Ki} \left(\frac{1}{2} e^2\right) \qquad (4)$$

$$= -\alpha e \frac{\partial e}{\partial Ki} \qquad (5)$$

Because

$$e = ym - yp \qquad (6)$$

and

$$\frac{\partial ym}{\partial Ki} = 0 \qquad (7)$$

it follows from (5) and (6) that

$$\frac{dKi}{dt} = a\, e \frac{\partial yp}{\partial Ki} \qquad (8)$$

The sensitivity coefficient $\frac{\partial yp}{\partial Ki}$ can easily be generated by a so-called sensitivity model.
Let the process be described by the differential equation

$$\frac{d^n yp}{dt^n} + K_{n-1} \frac{d^{n-1} yp}{dt^{n-1}} + \dots + K_1 \frac{dyp}{dt} + K_0 yp = u \qquad (9)$$

Differentiation of this equation with respect to Ki yields the differential equation of the sensitivity model

$$\frac{d^n}{dt^n} \frac{\partial yp}{\partial Ki} + K_{n-1} \frac{d^{n-1}}{dt^{n-1}} \frac{\partial yp}{\partial Ki} + \dots + K_0 \frac{\partial yp}{\partial Ki} = \frac{d^i yp}{dt^i} \qquad (10)$$

Apart from the input signals both differential equations (9) and (10) are identical. The output of eqn. (10) is the required sensitivity coefficient. Because the coefficients of eqn.s (9) and (10) are in general unknown or variable, the sensitivity coefficient $\frac{\partial yp}{\partial Ki}$ may be approximated by $\frac{\partial ym}{\partial Ki,m}$, which only requires knowledge and availability of coefficients and signals from the reference model.
A major disadvantage of the sensitivity model approach is that the stability of the total system cannot be analytically proved. Simulation experiments have to demonstrate the system's stability and should indicate the border of stability.

Example

A simple example will clarify the procedure. Let the process be described by the transfer function

$$H(s) = \frac{b_p'}{s^2 + a_p' s + 1} = \frac{Y(s)}{U(s)} \qquad (11)$$

A reference model of the same order and structure is selected with constant parameters which give the desired response. The variable or unknown process parameters can be compensated by the controller gains Ka and Kb. This leads to the differential equation of process plus controller :

$$\frac{d^2 y}{dt^2} + a_p \frac{dy}{dt} + y = b_p u \qquad (12)$$

where

$$a_p = a_p' + K_a \qquad (13)$$

and

$$b_p = b_p' + K_b \qquad (14)$$

The sensitivity models follow from eqn. (12) :

$$\frac{d^2}{dt^2}\frac{\partial y}{\partial a_p} + a_p \frac{d}{dt}\frac{\partial y}{\partial a_p} + \frac{\partial y}{\partial a_p} = -\frac{dy}{dt} \quad (15)$$

$$\frac{d^2}{dt^2}\frac{\partial y}{\partial b_p} + a_p \frac{d}{dt}\frac{\partial y}{\partial b_p} + \frac{\partial y}{\partial b_p} = u \quad (16)$$

The parameter adjustment laws are thus

$$\dot{a}_p = \alpha\, e \frac{\partial y_p}{\partial a_p} \quad (17)$$

$$\dot{b}_p = \beta\, e \frac{\partial y_p}{\partial b_p} \quad (18)$$

or

$$\dot{a}_p = \alpha\, e \frac{\partial y_m}{\partial a_m} \quad (19)$$

$$\dot{b}_p = \beta\, e \frac{\partial y_m}{\partial b_m} \quad (20)$$

Assuming that variations in the controller parameters due to the adaptation are fast compared with variations in the process parameters it follows that

$$\dot{K}_a = \dot{a}_p \quad (21)$$

and

$$\dot{K}_b = \dot{b}_p \quad (22)$$

2.2. The stability concept

The automatic parameter adjustment, based on adjustment laws like eqn. (8), introduce multipliers into the system and make the total system essentially non-linear. Linear theories fail to prove stability.
In recent years Liapunov's stability theory for non-linear systems, which dates from 1892 [9], as well as hyperstability theory [11], have been successfully applied to design stable MRAS. Although the designs based on Liapunov's stability theory and on hyperstability theory follow different ways, the adaptive laws which are found are similar. Based on Liapunov's second method the adaptive laws are derived as follows.
Describe the process and the reference model in state variables

$$\dot{\underline{x}}_p = A_p \underline{x}_p + B_p \underline{u} \quad (23)$$

$$\dot{\underline{x}}_m = A_m \underline{x}_m + B_m \underline{u} \quad (24)$$

where the index p belongs to the process and the index m to the reference model, and

$$A_p = A_p' + K_a \quad (25)$$

$$B_p = B_p' + K_b \quad (26)$$

and where A_p', B_p' are the varying process parameters and K_a and K_b are the adjustable controller gains.
Subtracting eqn. (23) from eqn. (24) yields the error differential equation

$$\dot{\underline{e}} = A_m \underline{e} + A \underline{x}_p + B \underline{u} \quad (27)$$

where

$$\underline{e} = \underline{x}_m - \underline{x}_p \quad (28)$$

$$A = A_m - A_p \quad (29)$$

$$B = B_m - B_p \quad (30)$$

Equation (27) is a non-linear differential equation because of the adjustment of K_a and K_b according to equations like eqn. (8). In order to design a stable MRAS it is necessary to prove that $\underline{e} \to \underline{0}$ for $t \to \infty$; in other words $\underline{e} = \underline{0}$ must be a stable equilibrium. The most simple positive definite Liapunov function, which not only contains the state vector $\underline{e}$, but also the additional state vectors $\underline{a}$ and $\underline{b}$, which contain the non-zero elements of A and B, is

$$V = \underline{e}^T P \underline{e} + \underline{a}^T \alpha \underline{a} + \underline{b}^T \beta \underline{b} \quad (31)$$

where P is an "arbitrary" positive definite symmetric matrix and α and β are diagonal matrices with positive elements, which will be shown to determine the speed of adaptation. The system is asymptotically stable when

$$\dot{V} = dV/dt < 0 \quad (32)$$

Differentiation of eqn. (31) and substitution of eqn. (27) yields

$$dV/dt = -\underline{e}^T Q \underline{e} + 2\underline{e}^T P A \underline{x}_p + 2\dot{\underline{a}}^T \alpha \underline{a} + 2\underline{e}^T P B \underline{u} + 2\dot{\underline{b}}^T \beta \underline{b} \quad (33)$$

with

$$-Q = A_m^T P + P A_m \quad (34)$$

When Q is an arbitrary symmetrical positive definite matrix the system is asymptotically stable for $\underline{e}$ (ordinary stable for $\underline{a}$ and $\underline{b}$), when

$$\underline{e}^T P A \underline{x}_p + \dot{\underline{a}}^T \alpha \underline{a} = 0 \quad (35)$$

and

$$\underline{e}^T P B \underline{u} + \dot{\underline{b}}^T \beta \underline{b} = 0 \quad (36)$$

This yields the adjustment laws

$$\dot{a}_{ni} = -\frac{1}{\alpha_{ni}} \left(\sum_{k=1}^{n} p_{nk} e_k\right) x_i \quad (37)$$

$$\dot{b}_{ni} = -\frac{1}{\beta_{ni}} \left(\sum_{k=1}^{n} p_{nk} e_k\right) u_i \quad (38)$$

The elements p_{nk} of the P matrix are found after selecting Q and solving eqn. (34).

Example

The design procedure will be illustrated again with the process of eqn. (11). Let the process plus controller be described by

$$\dot{x}_1 = x_2 \quad (39)$$

$$\dot{x}_2 = -x_1 - (a_p' + K_a)\, x_2 + (b_p' + Kb)\, u \quad (40)$$

or

$$A_p = \begin{bmatrix} 0 & 1 \\ -1 & -(a_p'+K_a) \end{bmatrix} \text{ and } \underline{b}_p = \begin{bmatrix} 0 \\ b_p' + K_b \end{bmatrix} \quad (41)$$

A reference model with the same structure is chosen with

$$A_m = \begin{bmatrix} 0 & 1 \\ -1 & -1.4 \end{bmatrix} \text{ and } \underline{b}_m = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (42)$$

This yields

$$A = \begin{bmatrix} 0 & 0 \\ 0 & -1.4+(a_p'+K_a) \end{bmatrix} \text{ and } \underline{b} = \begin{bmatrix} 0 \\ 1-(b_p'+K_b) \end{bmatrix} \quad (43)$$

Because only $a_{22} \neq 0$ and $b_2 \neq 0$ the adjustment laws simplify into

$$\dot{a}_{22} = -\frac{1}{\alpha_{22}} (p_{21}e_1 + p_{22}e_2)\, x_2 \quad (44)$$

$$\dot{b}_2 = -\frac{1}{\beta_2} (p_{21}e_1 + p_{22}e_2)\, u \quad (45)$$

The elements p_{21} and p_{22} of the matrix P are found by solving P from eqn. (34) after selecting an arbitrary positive definite matrix Q. Select for example

$$Q = \begin{bmatrix} 4 & 0.8 \\ 0.8 & 1.6 \end{bmatrix} \quad (46)$$

This yields $p_{21} = 1$ and $p_{22} = 1$.

After some searching it may also be possible to find a positive definite matrix Q belonging to a candidate matrix P.

From eqn. (43) it follows that

$$a_{22} = -1.4 + (a_p' + K_a) \quad (47)$$

Under the assumption that the controller parameter adjustment is fast compared with variations in the process parameters it follows that

$$\dot{K}_a = \dot{a}_{22} \quad (48)$$

and similarly that

$$\dot{K}_b = -\dot{b}_2 \quad (49)$$

Thus

$$K_a = -\frac{1}{\alpha_{22}} \int_0^t (e + \dot{e})\, x_2\, d\tau + K_a(0) \quad (50)$$

$$K_b = +\frac{1}{\beta_2} \int_0^t (e + \dot{e})\, u\, d\tau + K_b(0) \quad (51)$$

Remarks

– The adaptive laws derived above are sometimes referred to as integral adaptive laws because of the integration in equations (50) and (51). By choosing another Liapunov function the proportional plus integral adaptive law may be found, which may increase the speed of adaptation but also augments the sensitivity for noise. This leads, for instance, to

$$K_a = K_a(0) - \alpha'(e+\dot{e})\, x_2 - \frac{1}{\alpha_{22}} \int_0^t (e+\dot{e})\, x_2\, d\tau \quad (52)$$

– The adaptive gains α and β may be chosen arbitrarily; stability is guaranteed. However, smooth parameter adjustment requires some care with tuning of the adaptive gains.

– Comparing the adjustment law (19) with, for instance, (44) shows the following differences between the sensitivity approach and the stability approach : Instead of using only the signal e, the stable adaptive laws use the expression

$$p_{21}e + p_{22}\dot{e} \quad (53)$$

The derivative term has a positive effect on the system's stability. It can even be shown, by applying hyperstability theory, that the total phase lag, irrespective of the order of the system, is never larger than 90 degrees due to these derivative terms.

Instead of the sensitivity coefficient $\frac{\partial y}{\partial a}$, the state x_2 is used in the stable adaptive law (50). Comparison of both signals for one particular parameter show that the shape of these signals is similar, but that there is a phase lag in the sensitivity coefficient, which further deteriorates the system's stability.

– Because the choice of Q is arbitrary and the method gives only sufficient conditions for stability (they need not be necessary), there is some freedom in varying the elements of P to obtain an optimum performance.

3. IDENTIFICATION AND ADAPTIVE STATE ESTIMATION

When the process and the reference model are interchanged, the adjustment laws derived in the foregoing can be used for identification as well. The parameters of an "adjustable model" have to be made equal to the parameters of the process.

It has already been mentioned that the adaptive system is guaranteed to be asymptotically stable with respect to $\underline{e}$, but only ordinarily stable with respect to $\underline{a}$ and $\underline{b}$. This would imply that correct estimates of the process parameters cannot be guaranteed. However, dV/dt can only be equal to zero when $\underline{e} = \underline{0}$, which implies that either $\underline{x}_m = \underline{x}_p$ or $\underline{x}_m = \underline{x}_p = \underline{0}$ and $\underline{u} = \underline{0}$.

It can be seen that when $\underline{u}$ is sufficiently excited the equilibrium $\underline{e} = \underline{0}$ can only be reached when $\underline{a} = \underline{0}$ and $\underline{b} = \underline{0}$. Because $\underline{e}$ is guaranteed to converge to zero this also implies convergence of $\underline{a}$ and $\underline{b}$ to zero and thus successful identification.
It can be shown (see Section 4) that noise on the process states does not lead to biased parameter estimates. The momentary values of the parameters may fluctuate but in the mean the parameter values are correct. By selecting a suitable speed of adaptation smooth and unbiased parameter estimates can be obtained.
The same structure can also be used for adaptive state estimation with good noise reduction. Because $\underline{e} \rightarrow \underline{0}$ the states of the adjustable model will converge to the process states. When the speed of adaptation is low, noise on the process states will hardly influence the states of the adjustable model.
In the literature extensions of this principle are given. A second adjustable model can be added, which allows fast identification of the process parameters, also in the presence of noise [10,11]. It is also possible to use a second adjustable model, especially to improve the state estimation. This allows adaptation of the amount of filtering depending on the level of the noise [3].

4. APPLICATION OF MRAS IN PRACTICE

Until now it has been assumed that
1) process and reference model are of the same order and structure;
2) there are no nonlinearities;
3) there are no stochastic variations in the states of the process or its inputs.

In practice, none of these assumptions will be completely true. In this section a few solutions will be given to solve these problems.
Ad 1)
It has been shown [2] that structural differences between process and reference model are not disastrous as long as the reference-model structure contains the major process dynamics and simple adjustment rules are used. The adjustment laws (37) and (38) are such simple laws. More sophisticated algorithms, which give a better performance in the ideal case, fail when only small differences between the structures of process and reference model are present.
Ad 2)
Nonlinearities in the process (or in the reference model) can partly be treated in a way similar to the structural differences. Nonlinearities which can be considered as variations in the parameters of a simplified model will be compensated for by adjustment of the controller parameters. This requires a relatively high speed of adaptation.
More problems are caused by nonlinearities of the saturation type. This type of nonlinearity is quite common to amplifiers, in valves which are completely open or closed, et cetera. In principle there are two possibilities for dealing with this problem :
– switching off the adaptation as long as the element is saturated;
– modifying the input signal of process and reference model so that no saturation will occur.

The latter method has been successfully applied in an adaptive autopilot for ships [3,6]. No modifications of the adaptive loop itself are required and the proof of stability also remains unchanged. The non-linear element is in fact removed from the control loop.
Ad 3)
Noisy signals, due to disturbances or noisy measurements, are a problem when MRAS is applied for adaptation. This can easily be demonstrated. The noisy process states $\tilde{\underline{x}}_p$, and thus the error signal $\tilde{\underline{e}}$, can be denoted as follows :

$$\tilde{\underline{x}}_p = \underline{x}_p + \underline{\sigma}_x \qquad (54)$$

$$\tilde{\underline{e}} = \underline{e} + \underline{\sigma}_e \qquad (55)$$

where $\underline{\sigma}_x$ and $\underline{\sigma}_e$ are supposed to be stochastic signals with zero mean.
Because

$$\underline{e} = \underline{x}_m - (\underline{x}_p + \underline{\sigma}_x) \qquad (56)$$

it follows that

$$\underline{\sigma}_e = -\underline{\sigma}_x \qquad (57)$$

With the adjustment laws (37) and (38) this leads to expressions which contain, for instance,

$$\tilde{e}_i \tilde{x}_{ip} = e_i x_{ip} + e_i \sigma_{i,x} + e_i \sigma_{i,e} + \sigma_{i,e}\sigma_{i,x} \qquad (58)$$

Besides the desired term $e_i x_{ip}$, eqn (58) contains two terms with zero mean (because $\underline{\sigma}$ has a zero mean) and one term with non-zero mean :

$$\sigma_{i,e}\sigma_{i,x} = -\sigma^2_{i,x} \qquad (59)$$

Integration of this term will lead to controller parameters which drift away when the input is not sufficiently excited.
This problem is not met when MRAS is applied for identification. Because the model states will be noise-free there will be no cross-correlation term.
Measures which can be taken to suppress the influence of the noise are :
– switching off the adaptation when there are no set point changes;
– this can be achieved more smoothly by multiplying the adaptive gains by

$$\frac{1}{1+T} \qquad (60)$$

where T denotes the time after the last set point change;
– using $\underline{x}_m$ instead of $\underline{x}_p$ in equation (37). Although theoretically not correct, this approach gives good results in practice;
– using $\hat{\underline{x}}_p$ instead of $\underline{x}_p$, where $\hat{\underline{x}}_p$ is obtained from an adaptive state estimator as described in the former section;
– the terms $\sigma_{i,e}$ and $\sigma_{i,x}$ can be on-line estimated and

their products can be used to compensate for the drift caused by the terms of eqn. (59);
- a small dead band in the adaptive loop is also very effective, although the problem remains of defining as sharply as possible its width. The use of noise reduction filters is also possible. Care has to be taken for the destabilizing effect of these filters.

In a particular application a choice has to be made of one or a combination of these measures.

5. REALIZATION OF MRAS

More sophisticated control systems can most easily be implemented with digital hardware. In the former sections the continuous time approach has been followed. Assuming that the process itself is a continuous system, there are two ways to attack the design of discrete MRAS.

1. Use the continuous time algorithms and choose a sampling interval which allows the discretization to be neglected.
2. Describe the system in discrete form and derive discrete adjustment laws.

The first approach requires no further explanation. The second approach has been described by several authors. Landau has described discrete MRAS based on hyperstability theory [13,14].
The method of Liapunov, used in the foregoing, can be used as well, but requires a small modification in the structure. This is illustrated in fig. 5.
This structure is called series parallel because the reference model is partly in series and partly parallel with the process. This leads to the following equations :

$$\underline{x}_m(k+1) = A_m \underline{x}_p(k) + B_m \underline{u}(k) \tag{61}$$

$$\underline{x}_p(k+1) = A_p \underline{x}_p(k) + B_p \underline{u}(k) \tag{62}$$

where A_m, A_p, B_m and B_p are obtained after transforming the continuous time equations (23) and (24). Note the use of $x_p(k)$ instead of $x_m(k)$ in eqn (61).
It follows that

$$\underline{e}(k) = \underline{x}_m(k+1) - \underline{x}_p(k+1) \tag{63}$$

$$\underline{e}(k+1) = A(k)\underline{x}_p(k) + B(k)\underline{u}(k) \tag{64}$$

with

$$A(k) = A_m(k) - A_p(k) \tag{65}$$

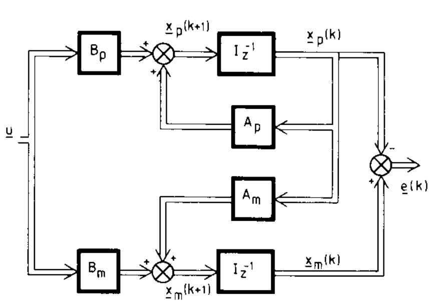

Fig. 5. Discrete series parallel MRAS

$$B(k) = B_m(k) - B_p(k) \tag{66}$$

A Liapunov function V(k) is defined :

$$V(k) = \underline{e}^T(k)P\underline{e}(k) + \underline{a}^T(k)\alpha\underline{a}(k) + \underline{b}^T(k)\beta\underline{b}(k) \tag{67}$$

where $\underline{a}$, $\underline{b}$, α and β are defined similarly to eqn. (31).
Because

$$\Delta V(k) = V(k+1) - V(k) \tag{68}$$

it follows that

$$\Delta V(k) = -\underline{e}^T(k)P\underline{e}(k) + \underline{e}^T(k+1)PA(k)\underline{x}_p(k)$$
$$+ [\underline{a}(k+1)+\underline{a}(k)]^T\alpha[\underline{a}(k+1)-\underline{a}(k)] + \underline{e}^T(k+1)PB(k)\underline{u}(k)$$
$$+ [\underline{b}(k+1)+b(k)]^T\beta[\underline{b}(k+1)-\underline{b}(k)] \tag{69}$$

The first term of eqn. (69) being negative definite (when P is positive definite) it follows that $\Delta V(k)$ is negative definite when

$$\Delta a_i(k) = a_i(k+1) - a_i(k) = -\frac{1}{\alpha_i}\left[\sum_{l=1}^{n} p_{nl}\, e_l(k+1)\right] x_{ip}(k) \tag{70}$$

$$\Delta b_i(k) = b_i(k+1) - b_i(k) = -\frac{1}{\beta_i}\left[\sum_{l=1}^{n} p_{nl}\, e_l(k+1)\right] u_i(k) \tag{71}$$

Compared with continuous MRAS the following has changed :
- Because $\underline{x}_m(k+1)$ is only a function of $\underline{x}_p(k)$ and not of $\underline{x}_m(k)$ the differences between $\underline{x}_m(k)$ and $\underline{x}_p(k)$ remain small. They can only diverge during one sample. This allows a high adaptive gain. In the presence of noise this advantage disappears because noise limits the adaptive gains.
- Although not theoretically supported, it has been shown that even better results can be obtained by making the states of the model and the states of the process not equal to each other at every sample, but only, for instance, once every five samples. This combines the good properties of parallel and series parallel systems [7,8].
- The advantage of unbiased parameter estimates which is realized by applying a parallel MRAS structure for identification is lost in the case of series parallel MRAS. It can easily be seen that there is cross correlation of the noise on the process states and noise on the error, both for adaptation and identification. Landau has shown that the algorithms which are found with the structure of fig. 5 are similar to the algorithms obtained by applying the least squares method [13].

6. CONCLUSIONS

Algorithms have been derived for continuous as well as for discrete MRAS. Because, strictly speaking, these algorithms are only valid in an idealized case, it was also necessary to consider the consequences of
- structural differences between process and reference model

– nonlinearities
– noise.

Although for some of the solutions which have been given the proof of stability is not valid anymore, experience has shown that they work in practice. At the Control Laboratory some successful applications have been realized :

– an adaptive autopilot for ships [1,2,3,6];
– attitude control of a satellite [7];
– speed control of a Ward Leonard system with varying load [8];
– adaptive control of the load frequency control in a power plant [5,12].

In the literature several other applications can be found.

7. LITERATURE

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