DIGITAL MODEL REFERENCE ADAPTIVE CONTROL WITH APPLICATIONS TO SHIP'S STEERING

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Abstract. The technique of Model Reference Adaptive Systems (MRAS) is applicable for direct adaptation of the parameters of a controller as well as for identification and state estimation. Originally the algorithms were mainly in continuous form, but presently digital implementation is common. Discrete MRAS can be designed from two different points of view. When the sampling interval is small enough the contiuous time equations can easily be transformed to discrete equations. Deriving the discrete algorithms directly leads to totally different structures. A combination of the algorithms which are found by these two approaches yields adaptive laws with improved convergence.
This paper describes the consequences of application of MRAS in practice. As a typical test case an adaptive autopilot for course control of ships has been designed. It is shown how to deal with certain types of non-linearities in the process' dynamics and the hydraulic actuators. Without special precautions the performance of MRAS in a noisy environment is bad. Several solutions are suggested, which can easily be implemented in a digital computer.
The resulting adaptive control system has been tested during full scale trials with various ships.

Keywords. Adaptive control; identification; state estimation; digital computer applications; ships.

1. INTRODUCTION

In recent years adaptive control has got a lot of attention. Among the various approaches the technique of model reference adaptive control systems (MRAS) is an important one. MRAS has a dual character. It can be used for direct adaptation of the controller gains, without explicit identification, as well as for identification of the parameters of an unknown process. In the latter case adaptive state estimation is simultaneously obtained. In early publications about MRAS, mainly the sensitivity concept is used to derive the adaptive laws. In recent years designs based on stability theory are favourite. (Landau,1974).Originally the algorithms were mainly in a continuous form. Presently digital implementation of the adaptive controller is common and discrete algorithms are at hand.(Landau,1979). In this paper also a combination of both approaches will be described.

The paper is organized as follows. In section 2 the basic algorithms are given for the idealized situation. Modifications which are required in order to make the theory applicable in practice will be discussed in section 3. In section 4 the proposed solutions will be illustrated with the design of an adaptive autopilot for steering of ships, where MRAS is applied for direct adaptation of the controller gains as well as for identification and adaptive state estimation.

2. ADAPTIVE LAWS FOR THE IDEALIZED CASE

In an idealized situation the process can be exactly described by a linear differential equation of known order. The desired behaviour of this process is given by a parallel reference model of the same order and structure as the process. The differences between the states of the process and the reference model are used in the adaptive controller to adjust the gains of the controller of the process. When the process is described by

$$\underline{\dot{x}}_p = A_p \underline{x}_p + B_p \underline{u} \tag{1}$$

and the reference model by

$$\underline{\dot{x}}_m = A_m \underline{x}_m + B_m \underline{u} \tag{2}$$

the differential equation of the error $\underline{e}$ is found by subtracting (1) from (2):

$$\underline{\dot{e}} = A_m \underline{e} + A \underline{x}_p + B \underline{u} \tag{3}$$

where

$$\underline{e} = \underline{x}_m - \underline{x}_p \tag{4}$$

$$A = A_m - A_p \tag{5}$$

$$B = B_m - B_p \tag{6}$$

Following the stability approach, the adaptive laws are found from the proof of stability of the differential equation (3). The proof of stability can be given with the aid of the hyperstability concept or by applying Liapunov's second method. Both methods yield similar adaptive laws. The proof according to Liapunov starts with choosing a positive definite Liapunov function V. A suitable Liapunov function is

$$V = \underline{e}^T P\, \underline{e} + \underline{a}^T \alpha\, \underline{a} + \underline{b}^T \beta\, \underline{b} \tag{7}$$

where

- P = a positive definite matrix
- $\underline{a}$ = a vector which contains the non-zero elements of the A matrix
- $\underline{b}$ = a vector which contains the non-zero elements of the B matrix
- α, β are diagonal matrices with positve coefficients.

Stability of the system is guaranteed when dV / dt is negative definite. This leads to the so called integral adaptive laws:

$$a_i(t) = - \frac{1}{\alpha_i} \int_o^t \left(\sum_{\ell=1}^{n} p_{n\ell}\, e_\ell \right) x_{p,i}\, d\tau + a(0) \tag{8}$$

$$b_i(t) = - \frac{1}{\beta_i} \int_o^t \left(\sum_{\ell=1}^{n} p_{n\ell}\, e_\ell \right) u_i\, d\tau + b(0) \tag{9}$$

where n is the order of the process and p_n are elements of the P matrix. (The system is supposed to be noted in phase variable form; otherwise a slightly more complicated formula is found). The elements of the P matrix are found after solving

$$A_m^T P + P A_m = -Q \tag{10}$$

where Q is an arbitrary symmetrical positive definite matrix. Under the assumption that the speed of adaptation is high with respect to the speed of parameter changes, it follows that

$$\dot{K}_{a,i} = - \dot{a}_i \tag{11}$$

$$\dot{K}_{b,i} = - \dot{b}_i \tag{12}$$

where $K_{a,i}$ and $K_{b,i}$ are the adjustable controller gains of the process.

The dual character of MRAS has already been mentioned. When process and reference model are interchanged, that means that the process takes the place of the reference model, while the process is replaced by an adjustable model the same structure can be used for identification. This gives the adjustment laws:

$$a_i(t) = - \frac{1}{\alpha_i} \int_o^t \left(\sum_{\ell=1}^{n} p_{n\ell}\, e_\ell \right) x_{m,i}\, d\tau + a_i(0) \tag{13}$$

$$b_i(t) = - \frac{1}{\beta_i} \int_o^t \left(\sum_{\ell=1}^{n} p_{n\ell}\, e_\ell \right) u_i\, d\tau + b_i(0) \tag{14}$$

The only difference with equations (8) and (9) is that $x_{p,i}$ has been replaced by $x_{m,i}$.

- Besides parameter estimation also adaptive state estimation is obtained, because when $\underline{e} \to \underline{0}$ the states of process and reference model are equal.

- The Liapunov function guarantees asymptotic stability with respect to $\underline{e}$, but only ordinary stability for $\underline{a}$ and $\underline{b}$. To ensure that $\underline{a} \to \underline{0}$ and $b \to \underline{0}$ the system must be sufficiently excited.
- More complex adaptive laws can be found by choosing other Liapunov functions. In section 3 it will be discussed however, that for practical applications the given adaptive law suffices.

When a digital computer is used to implement the adaptive controller the discrete character of the controller has to be taken into account. As long as the sampling interval is sufficiently small, the continuous time equations, which were derived before, remain valid. Another approach is to describe the continuous system in discrete form and to derive discrete adaptive laws. Using the Liapunov approach the proof of stability for discrete systems can only be given for the so called series parallel structure. (Fig. 1) .

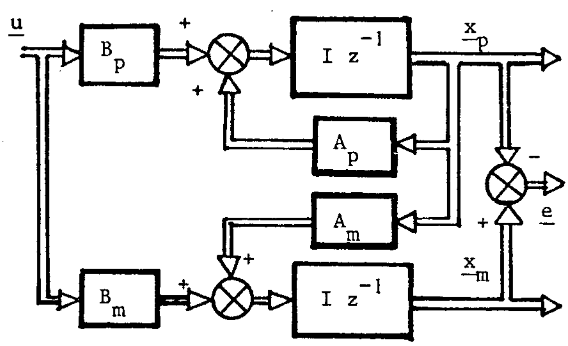

Figure 1. Discrete series parallel MRAS

When the process is described by

$$\underline{x}_p(k+1) = A_p\, \underline{x}_p(k) + B_p\, \underline{u}(k) \tag{15}$$

and the reference model by

$$\underline{x}_m(k+1) = A_m\, \underline{x}_m(k) + B_m\, \underline{u}(k) \tag{16}$$

the differential equation of the error is

$$\underline{e}\,(k+1) = A(k)\, \underline{x}_p(k) + B(k)\, \underline{u}(k) \tag{17}$$

with

$$\underline{e}\,(k) = \underline{x}_m(k) - \underline{x}_p(k) \tag{18}$$

The Liapunov function is selected

$$V(k) = \underline{e}^T(k)P\underline{e}(k) + \underline{a}^T(k)\alpha\underline{a}(k) + \underline{b}^T(k)\beta\underline{b}(k) \qquad (19)$$

The coefficients in these equations are defined in a similar way as for the continuous case. To proof the stability of this system

$$\Delta V(k+1) = V(k+1) - V(k) \qquad (20)$$

has to be negative definite. This leads to the adjustment laws

$$\Delta a_i(k+1) = -\frac{1}{\alpha_i}\ \{\sum_{\ell=1}^{n} p_{n\ell}e_\ell(k+1)\ x_{p,i}(k)\} \qquad (21)$$

$$\Delta b_i(k+1) = -\frac{1}{\beta_i}\ \{\sum_{\ell=1}^{n} p_{n\ell}e_\ell(k+1)\ u_i(k)\} \qquad (22)$$

The speed of adaptation can considerably be improved when a combination of the continuous time and discrete time algorithms is used. This idea is based on the following philosophy.

The main goal of the adaptive system, to bring $\underline{e} \to \underline{0}$ is realized by bringing $\underline{a} \to \underline{0}$ and $\underline{b} \to \underline{0}$. However, when the speed of adaptation is high, it may happen that at a certain moment $\underline{a} = \underline{0}$ and $\underline{b} = \underline{0}$, while due to the system dynamics $\underline{e} \neq \underline{0}$. The "adjustment" of $\underline{a}$ and $\underline{b}$ will then continue which leads to oscillatory parameters. This continuing "adjustment" can be stopped in time, when at regular time intervals $\underline{x}_m$ is made equal to $\underline{x}_p$. Although the series parallel system carries out this principle in the most extreme way, it has turned out that better results are obtained when the updating takes place only every 5^{th} or 20^{th} sample. (Van den Bosch and Jongkind, 1980 and section 4). This is probably due to the fact that pure series parallel MRAS is extremely sensitive for noise, especially when high adaptive gains are applied.

3. PROBLEMS IN PRACTICAL APPLICATIONS

When the theory of the former section has to be applied in practice this cannot always be done straightforwardly. Several assumptions were implicitly or explicitly made.

- Process and reference model were supposed to be of the same order and structure.
- Process and reference model were supposed to be linear.
- There are no disturbances (that means no noise on the state variables)
- All the states of process and reference model were supposed to be measurable.

In practice this idealized situation is hardly never met. In this section solutions are suggested how to deal with these problems.

Noise

Noise has two effects. Firstly it causes (undesirable) fluctuations of the parameters. These fluctuations can be kept small by choosing low adaptive gains. The other effect is more serious, because it causes a continuous drift in the parameter values. Suppose $\underline{x}_p$ is corrupted with an additive noise signal $\underline{\sigma}$ with zero mean. In the adaptive law elements of $\underline{x}_p$ are multiplied with elements of $\underline{e} = \underline{x}_m - \underline{x}_p$. This leads to a term

$$e_i x_{p,i} = \{x_{m,i} - (x_{p,i} + \sigma)\}.(x_{p,i} + \sigma) \qquad (23)$$

Besides the desired term $e_i x_{p,i}$ and two terms with zero mean, eqn (23) contains a term σ^2 which has no zero mean. This term causes the parameter drift. Note that in the case of identification unbiased parameters will be obtained, because $\underline{x}_p$ is then replaced by $\underline{x}_m$. This advantage is lost however, when the series parallel configuration is used for identification.

To prevent drift of the parameters one or more of the following measures may be taken. Especially when the adaptive system is implemented digitally these measures are easy to realize.

- The adaptation is switched off when there are no setpoint changes.
- This may be done more smooth by multiplying the adaptive gains with

$$\frac{1}{1 + T} \qquad (24)$$

  where T is the time after the last setpoint change. (so called decreasing adaptive gains).
- Instead of using $\underline{x}_p$ in the adaptive laws, as an approximation $\underline{x}_m$ may be used.
- A small dead zone may be used ($> \sigma^2$) in the adaptive loop.
- When the variance of the noise σ^2 is measured on-line, it may be subtracted from the output of the adaptive algorithms.

Constant disturbances in the process (or the mean value of noise with non-zero mean) can be compensated for by designing an adaptive integral action. This constant disturbance is modelled as an additional input with unknown gain. It can be compensated by a constant signal in the controller output with the same amplitude but with opposite sign. This yields for example that one of the equations (9) becomes:

$$K_c(t) = -\gamma \int_0^t (\sum_{\ell=1}^{n} p_{n\ell}e_\ell)\ d\tau + K_c(0) \qquad (25)$$

Of course this alternative integrating action can also be used for slowly varying disturbances.

Structural differences and non-linearities

In practice the situation that process and reference model have the same order and structure will seldom be found. The order of the refrence model will always be an approximation of the real order of the process. The same holds for the structure. There may also be non-linearities in the process or even in the reference model. Experiments with an adaptive autopilot for ships (Van Amerongen et. al. 1975) have shown that in this case simple algorithms perform much better than more complex algorithmswhich require more equality between the structure of the process and the reference model. Further it appears that those non-linearities in the process which can be seen as a modification of one or more of the process parameters, can at best be neglected. When the adaptation is fast enough the adaptive gains will compensate for these variations.

Non-linearities caused by saturation effects however, for instance in actuators, cannot be simply neglected. Because the structure of process and reference model may differ it is also not possible to add a similar element in the reference model as well. Solutions to deal with this problem are

- Switch off the adaptation as long as the system is in saturation
- Add a series model which modifies the system's input in such a way that the saturation is eliminated in fact.

Both solutions are also applicable when the reference model contains a saturation element. The proof of stability of the system is not affected by these measures. An example of the second solution will be given in the next section.

4. DESIGN OF AN ADAPTIVE AUTOPILOT FOR SHIPS

To illustrate the ideas given in the former sections the design of an adaptive autopilot for ships will be discussed. (Van Amerongen and Van Nauta Lemke, 1979). The design starts with defining the desired performance and the manual settings which remain necessary to influence the performance. Based upon results of an inquiry among officers of the Royal Netherlands Navy and the Dutch Merchant Navy the following requirements have been formulated. (Van Amerongen and Van Nauta Lemke, 1978) .

- During course keeping, that is sailing a straight track, it must be possible to choose between maximum course keeping accuracy and maximum fuel economy. This can be expressed in a quadratic criterium of the form

$$J = \int_0^t (\varepsilon^2 + \lambda\,\delta^2)\,d\tau \qquad (26)$$

where ε is the heading error and δ is the rudder angle; λ is a weighting factor. Besides, it is important that relative high frequency rudder motions, caused by the waves which only give loss of speed, without having any positive effect on the course error, are prevented anyhow.

- During course changing the desired performance is described by the following demands:
- a clear start of the manoeuvre
- at the end of the manoeuvre there must be no overshoot
- between start and end of the manoeuvre there is phase of constant turning.

This is illustrated in Fig. 2.

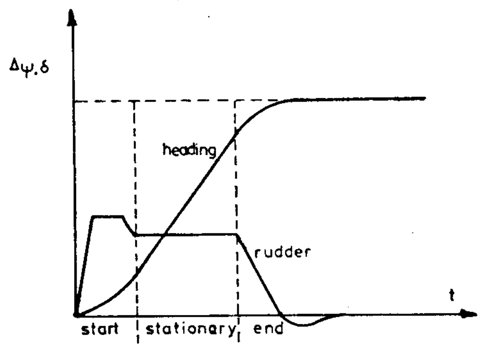

Figure 2. Course changing manoeuvre

Conventional autopilots mainly consist of a PID controller, extended with a rudder limit (to prevent undesirable large rudder angles) and an adjustable dead band (to prevent too much rudder motions in bad weather conditions) The settings of the PID controller do not have a clear and simple relation with the demands of the user. Fixed controller settings are not possible due to changing steering dynamics and variations in the required performance. MRAS is applied to this system in order to provide simple possibilities of adjustment to the user. The adjustment of the controller itself is carried out automatically.

The rudder angle is computed with the algorithm

$$\delta = K_p\varepsilon - K_d\dot{\psi} + K_i \qquad (27)$$

where

δ = the rudder angle
ε = the course error ($\psi_r - \psi_c$)
$\dot{\psi}$ = the rate of turn
K_i = the rudder off-set which will be computed with an algorithm of the form of eqn. (25).

During course changing K_p, K_d and K_i are adjusted by means of direct adaptation. When the influence of K_i is neglected, the process dynamics are approximated by:

$$\frac{\psi_c}{\psi_r} = \frac{K_p K_s/\tau_s}{s^2 + s(1 + K_d)/\tau_s + K_p K_s/\tau_s} \quad (28)$$

where

ψ_c = the actual heading

ψ_r = the desired heading

K_s, τ_s are parameters of a simplified model of the ship.

The reference model is selected:

$$\frac{\psi_m}{\psi_r} = \frac{K_{pm}/\tau_m}{s^2 + s/\tau_m + K_{pm}/\tau_m} \quad (29)$$

To bring the model dynamics in the range of the system's dynamics a good choice of the parameter τ_m appears to be

$$\tau_m \approx \tfrac{1}{2}\, \tau_{s,min} \quad (30)$$

where $\tau_{s,min}$ is the minimum value of τ_s which can be expected. K_{pm} is computed based upon the desired damping ratio. In order to be able to select a desired rate of turn, a rate of turn limit is added to this model as follows

$$-\dot{\psi}_{max} < \dot{\psi}_m < \dot{\psi}_{max} \quad (31)$$

where $\dot{\psi}_m$ is the rate of turn signal of the reference model and $\dot{\psi}_{max}$ is the desired rate of turn.

Because of the presence of this rate of turn limiter in the reference model and a rudder limit in the hydraulic steering machine (or an even more severe rudder limit in the autopilot itself) precautions have to be taken to ensure that these limiters do not detoriate the performance of the adaptive system. The solution with a series model which was suggested in the former section has been used.

This series model has the same structure as the reference model, but with the exception that the limiter in the series model is not only adjusted based upon the desired rate of turn, but also based on the rudder limit. Figure 3 shows the realization of this series model.

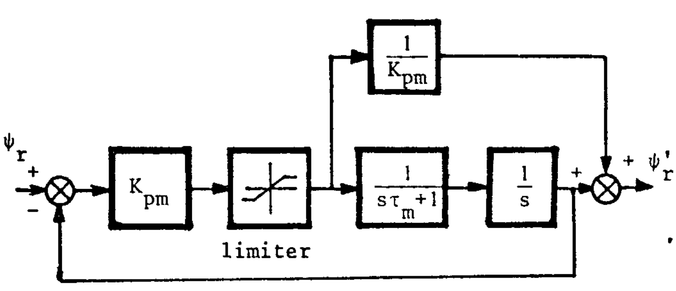

Figure 3. Series model to modify the setpoint

In this figure ψ_r denotes the reference heading; ψ'_r is the reference heading after modification by the series model.

In order to prevent parameter drift caused by noise (which is mainly caused by the influence of the waves on the ship's motions) in the adaptive law the model states are used instead of the states of the process. Also the principle of decreasing adaptive gains is used. This yields the adjustment laws:

$$K_p = K_p(0) + \beta \int_0^t (\psi'_r - \psi_m)(p_{12}e_1 + p_{22}e_2)\,d\tau \quad (32)$$

$$K_d = K_d(0) - \alpha \int_0^t \dot{\psi}_m \, (p_{12}e_1 + p_{22}e_2)\,d\tau \quad (33)$$

$$K_i = K_i(0) + \gamma \int_0^t 1 \, (p_{12}e_1 + p_{22}e_2)\,d\tau \quad (34)$$

where $\dot{\psi}_m, \psi_m$ are the rate of turn and heading of the parallel reference model and

$$e_1 = \psi_m - \psi_c$$

$$e_2 = \dot{\psi}_m - \dot{\psi}$$

Every 5 seconds the rate of turn and heading of the reference model are made equal to the the rate of turn and heading of the ship.

<u>Course keeping</u>

During course keeping K_p and K_d are computed based on the ship's parameters K_s and τ_s and on the criterion (26). This yields

$$K_p = 1 / \sqrt{\lambda} \quad (35)$$

$$K_d = 1 / K_s\{\sqrt{(1 + 2K_s\tau_s/\sqrt{\lambda})} - 1\} \quad (36)$$

The ship's parameters K_s and τ_s are found by an identification procedure which is also based on MRAS.

$$\frac{K_s}{\tau_s} = \frac{K_s}{\tau_s}(0) - \beta' \int_0^t (\delta - K_{i,m})\, e\, d\tau \quad (37)$$

$$\frac{1}{\tau_s} = \frac{1}{\tau_s}(0) + \alpha' \int_0^t \hat{\dot{\psi}}\, e\, d\tau \quad (38)$$

$$K_{i,m} = K_{i,m}(0) + \gamma' \int_0^t 1\, e\, d\tau \quad (39)$$

where

$\hat{\dot{\psi}}$ = the output of the adjustable model and

$e = \hat{\dot{\psi}} - \dot{\psi}$

Because $\hat{\dot{\psi}}$ is a noise free estimate of $\dot{\psi}$, $\hat{\dot{\psi}}$ is used in the controller instead of $\dot{\psi}$. This considerably reduces the number of rudder motions in bad weather conditions which has a positive effect on the fuel economy.

In (Van Amerongen and Van Nauta Lemke, 1979) it is indicated how this state estimation can be improved by adding a second adjustable model to this system. This second adjustable model is more or less comparable with a Kalman filter. It can be made adaptive with respect to the level and characteristics of the noise.

5. RESULTS AND CONCLUSIONS

The proposed autopilot system has been implemented into a small digital computer (DECLAB 11/03) and it was tested on board various ships. The course changing behaviour was tested with a series of standard course changes programmed in the computer. With varying ship's speeds and several sea-states the adaptive autopilot performes well. The type of response defined in figure 3 is realized.

During course keeping the number of rudder movements is considerably reduced compared with a (well adjusted) conventional autopilot or manual steering. Figure 4 illustrates this. This reduction of rudder motions can be obtained because of the MRAS-based state estimation. Speed measurements carried out during these trials indicate that the mean speed is about 1 percent higher when the adaptive autopilot controls the ship. Additional trials carried out in a towing tank indicate savings between 0.5 and 5 percent. Figure 4 shows the course error ε, the rate of turn signal $\dot{\psi}$, the estimated rate of turn $\hat{\dot{\psi}}$, the rudder angle δ and the criterion (26) when $\lambda = 10$. Steering with the adaptive autopilot is denoted as ASA.

These succesful trials indicate that MRAS is applicable, also in situations with non-linear systems and noisy measurements. The reduced number of controller settings, which have a clear meaning to the user, is a great improvement. The fuel savings which can be obtained are of increasing importance in this time of rapidly increasing fuel prices.

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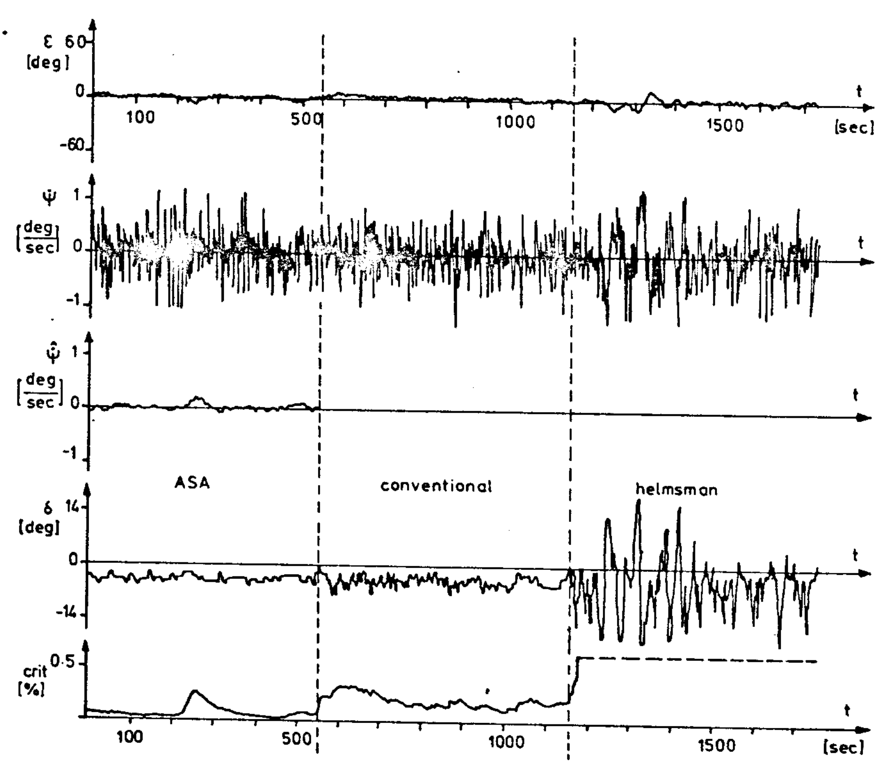

Figure 4. Comparison between the adaptive autopilot ASA, a conventional autopilot and the helmsman.