Criteria for Optimum Steering of Ships

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Abstract

During the last few years an increasing number of papers on the optimum design of autopilots for ships has been published. Several new control techniques have been used to design adaptive autopilots which automatically adjust their parameters. One of the problems which has not yet been completely solved is the definition of a criterion for optimum steering. Full scale measurements, where the increased performance is measured directly in terms of greater speed or less fuel consumption, have hardly been reported in the past.

This paper discusses the problem of optimum steering, not only from the point of view of economics, but also from the user's point of view. The character of the disturbances is also taken into account. A practical solution for the adjustment of autopilots, combining the philosophies of several approaches to this problem is suggested.
1. Introduction

Recent developments in control theory and the availability of small and inexpensive digital computers have led to an increasing interest in the design of autopilots for ships. Conventional autopilots basically consist of a PID controller with additional non-linear elements such as a rudder limit and a dead band. Because of the great number of settings which interact with each other, manual adjustment of these autopilots is difficult. This is also due to the fact that no simple criterion for optimum steering, which is valid under all circumstances, is at hand.

Today adaptive control techniques which enable automatic tuning of the parameters of the controller itself are available. The settings which remain to be adjusted by the user can be smaller in number and may have a more clear meaning than the settings of a conventional autopilot. However, for automatic adjustment of the parameters the availability of a criterion which defines optimum steering is still essential.

In areas with a high traffic density and in confined waters good steering is mainly related to accuracy. Easier controller adjustment which can be achieved by applying adaptive control will certainly contribute to the safety. A totally different situation is met on the ocean, where minimization of the drag rather than accurate steering is the main purpose. The recent increase in fuel prices makes this aspect of growing importance.

In the literature several successful attempts to design an adaptive autopilot have been reported (See for instance Schilling, 1976; Källström et al., 1979; Reid and Williams, 1978; Ohtsu et al., 1979; Van Amerongen and Van Nauta Lemke, 1978, 1979). Although all authors claim to have developed an optimum adjusted autopilot, they do not agree upon the definition of the performance index which should be used to obtain optimum steering.
Most designs are based on a quadratic criterion where course errors are weighted against rudder motions. Originally, this criterion was suggested by Koyama (1967). Norrbom (1972) proposes a similar criterion, but by following a different approach he finds other numerical values which define optimum steering. In the following these two approaches will be discussed and a slightly more extensive criterion which also weights the ship's rate of turn will be introduced. The influence of the frequency of the disturbances with respect to this optimization problem will also be considered.

Fuel economy mainly plays a role during course keeping. During course changing other aspects determine the optimality of the response. In Section 2 the optimum course changing performance will be defined. Section 3 deals with course keeping. Results obtained from various experiments are dealt with in Section 4. The conclusions are summarized in Section 5.

2. Optimum course-changing

The conventional settings of an autopilot based on a PID controller hardly have any relation to what the user really wants to be adjustable. This thesis is based on the information obtained from an inquiry which was conducted among officers of the Royal Netherlands Navy and the Netherlands Merchant Navy (Prins, 1978). This inquiry as well as discussions in several meetings with the people who responded, indicate that the optimum course-changing manoeuvre can be described by Figure 1.

![Figure 1. Optimum course-changing manoeuvre.](image-url)
In this manoeuvre three phases can be distinguished

- the start
- a stationary part
- the end

Because course changes in most situations are related to giving way to other ships, it is important that the start of the turn clearly shows the intention of the manoeuvre to other ships. Although this first phase might require adjustment of the controller settings, from the user's point of view there is no need for any special setting.

In the second phase, the turn is stationary. On conventional autopilots the only way to influence the behavior in this stage of the turn is with the aid of the rudder limit. However, the effect of a certain rudder limit depends, for instance, on the speed. Therefore, it might be desirable to select instead an adjustable rate of turn or an adjustable turning-circle diameter.

The end of the manoeuvre is mainly determined by the demand that there must be no overshoot. Again, this is important in order to clearly show other ships what is going on. No user-adjustable settings are necessary in this phase. The only setting which is required from the user's point of view is a setting which defines the stationary part of the manoeuvre. Internal controller adjustment which is required to realize the whole desired response should be automatized. In Van Amerongen and Van Nauta Lemke (1978, 1979) it is indicated how this can be realized by applying model-reference adaptive control.
3. Optimum course keeping

During course keeping the optimum performance has to be defined in a different way. Two extreme situations can be distinguished. When sailing in confined waters the main purpose is accurate course control. The safety aspects, which require accurate steering in confined waters, do not dominate in the ocean anymore. In that case the main purpose is to cover the distance to the destination in minimum time (assuming constant thrustpower). The fuel consumption is then minimum as well. Neither the demand for accurate steering nor the demand for economy can be easily translated into controller settings.

The inquiry mentioned above indicates the user's preference for one setting which enables him to select between maximum accuracy and maximum fuel economy (if these two modes would indeed require different controller settings). This implies again that adjustment of the conventional controller settings has to be automatized.

To study this problem a mathematical model of the ship's steering dynamics is required. For course keeping with course-stable ships, the first-order Nomoto model gives a good description of the steering dynamics (Nomoto, 1957). Without special precautions the desired rudder angles may be large when the ship sails in rough weather. Then this means that the dynamics of the steering machine cannot be neglected anymore. In this case the block diagram of Figure 2 can be used to describe the process.

![Block Diagram](image)

**Figure 2. Simplified steering dynamics for course keeping**
In this figure the following variables are used

\[ \delta_r, \] the desired rudder angle (the output of the autopilot)
\[ \delta, \] the actual rudder angle
\[ \dot{\psi}, \] the rate of turn
\[ \psi, \] the heading
\[ \psi_r, \] the reference heading

The maximum rudder speed, although it will depend on the type of ship, is approximately between 2.5 and 7 degrees per second.

3.1. Accurate steering

In principle, high accuracy can be guaranteed by selecting high gains in the controller which may be described by

\[ \delta_r = K_p (\psi_r - \psi) - K_d \dot{\psi} \] (1)

The integrating action, required to compensate for constant disturbances, will be neglected in this stage. As long as the steering machine and further higher-order dynamics are neglected the proportional gain \( K_p \) can have any value, because a sufficiently stable system can be obtained by properly adjusting \( K_d \). For large and fast rudder motions, however, the steering machine will have a destabilizing effect. In practice, this limits the range of allowable gains, and especially in rough weather it introduces the need to suppress high-frequency components of the controller output, because these high-frequency movements have hardly any positive effect on the motions of the ship, due to the low-pass character of the ship's transfer function.

The strategy for accurate steering can thus be formulated as:

Use as high gains as possible with respect to stability and prevent high-frequency controller outputs which do not contribute to the steering accuracy but which instead enlarge the heading error.
3.2 Economical steering

The purpose of economical steering has been mentioned already: minimization of the fuel consumption under the assumption of constant thrustpower. Besides, prevention of wear and tear of the steering machine may play a role. In order to be able to adjust the controller, a relation between this criterion and the controller settings has to be derived. If it were possible to accurately measure the ship's speed and the thrustpower during normal operation, experimental optimization methods (e.g. hill climbing) could be applied; the controller gains would then be directly adjusted in relation to these variables.

To find an analytical expression for the controller gains the criterion should be a function of the state variables and input signals of the course-control system. Koyama (1967) has suggested the following criterion:

\[ J = \int_{0}^{t} (\varepsilon^2 + \lambda \delta^2) \, dt \]  

(2)

where \( \varepsilon \) denotes the heading error
\( \delta \) the rudder angle
and \( \lambda \) is a weighting factor.

The factor \( \varepsilon^2 \) translates the elongation of the distance due to course errors into a speed loss. Increased resistance due to steering also leads to loss of speed. This is expressed by the factor \( \lambda \delta^2 \), where \( \lambda \) is obtained after normalizing the coefficient of \( \varepsilon^2 \) on 1. When \( \varepsilon \) and \( \delta \) are in degrees the percentage of speed loss is

\[ J = \left( \frac{\pi}{180} \right)^2 \cdot \frac{1}{4} \cdot \frac{1}{T} \int_{0}^{T} (\varepsilon^2 + \lambda \delta^2) \, dt \]  

(3)

The weighting factor \( \lambda \) should be determined with the aid of full-scale trials or model tests. Koyama suggests values of \( \lambda \) of approximately 8 - 10.
Norrbin (1972) follows a different approach to calculate \( \lambda \). In fact, Norrbin totally neglects the contribution of course errors on elongation of the distance. Losses which are considered are the loss of speed due to increased resistance of the rudder itself (proportional with \( \delta^2 \)) and to the centrifugal force. The latter can be described by the term

\[
\dot{\psi} \sim \dot{\psi}^2
\]

(4)

where \( \psi \) is the drift velocity. Under the assumption that a ship under autopilot control will yaw periodically, the relation holds

\[
\ddot{\psi} = \omega \dot{\psi}
\]

(5)

where \( \omega \) is the frequency of yawing.

This leads to a criterion whose shape is similar to Eq. (2) but whose values of \( \lambda \) are much smaller, for instance about \( \lambda = 0.1 \).

By choosing criterion (2) and neglecting the steering machine in Figure 2, straightforward application of optimal control theory yields the controller gains \( K_p \) and \( K_d \) (See Eq. (1)).

\[
K_p = \frac{1}{\sqrt{\lambda}}
\]

(6)

\[
K_d = \frac{1}{K_s} \left\{ \sqrt{1 + \frac{2K_t S}{\sqrt{\lambda}}} - 1 \right\}
\]

(7)

This yields, with \( \lambda = 0.1 \) and \( \lambda = 10 \), respectively,

\[
K_p = 3.2 \quad \text{and} \quad K_d = 0.32
\]

while \( K_d \), being a function of the ship's dynamics, assures a sufficiently damped system.

Although the controller settings found for different values of \( \lambda \) are quite different, both authors agree on the point that the rate of turn should be kept as small as possible.
By applying Norrbin's criterion this should be accomplished by preventing any course error (and thus turning) by means of tight control. According to Koyama course errors themselves are not so serious, but large rudder angles which cause high turning rates must be prevented. Probably one of the major reasons for these different attitudes is that Koyama explicitly takes into account the nature of the disturbances. In bad weather Norrbin's criterion might result in rudder motions which would enlarge the ship's motions rather than reduce them. The power of the steering machine is too small to compensate for the high-frequency wave forces. This problem is most evident when steering small ships.

Therefore, in bad weather a value of $\lambda = 10$ will be a good choice. When there are no high-frequency disturbances or when steering large ships, which are hardly affected by these disturbances, $\lambda = 0.1$ may be an optimum choice. By choosing a high value of $\lambda$, and thus low controller gains, the whole course control system, in fact, gets a lower bandwidth and automatically filters high frequencies.

This filtering can also be done more explicitly. How the principles of model-reference adaptive control can be used to design an adaptive state estimator for this purpose has been described by Van Amerongen and Van Nauta Lemke (1978, 1979). By observing the ratio between high-frequency and low-frequency disturbances, the state estimation can be improved and can also be made adaptive with respect to the level of the disturbances.

This yields state variables which can be used in a slightly more extensive criterion, based on the following philosophy. Only the low-frequency course errors have to be compensated. Since these course errors are only important with respect to elongation of the distance, which is small compared to other losses, correction of the course error should be done by simultaneously minimizing the drag caused by the rudder itself and the drag caused by the centrifugal force.
\[ \hat{\psi} \chi \hat{\psi}^2 \]  

where \( \hat{\psi} \) denotes the rate of turn caused by steering (the components due to disturbances thus being removed).

This leads to the criterion

\[
J = \int_{0}^{T} \left( \varepsilon^2 + \lambda_1 \hat{\psi}^2 + \lambda_2 \delta^2 \right) \, d\tau
\]

From the data provided by Norrbin it can be computed that for two particular ships this yields the numerical values:

\[
J = \left( \frac{\pi}{180} \right)^2 \cdot \frac{1}{4} \cdot \frac{1}{T} \int_{0}^{T} \left( \varepsilon^2 + 15000 \hat{\psi}^2 + 8\delta^2 \right) \, dt
\]  

(for a tanker)

and

\[
J = \left( \frac{\pi}{180} \right)^2 \cdot \frac{1}{4} \cdot \frac{1}{T} \int_{0}^{T} \left( \varepsilon^2 + 1600\hat{\psi}^2 + 6\delta^2 \right) \, dt
\]  

(for a cargo liner)

Optimizing criterion (9) for the process of Figure 2 and neglecting again the steering-machine dynamics yields the controller gains

\[
K_p = \frac{1}{\sqrt{\lambda_2}}
\]

\[
K_d = \frac{1}{K_s} \left\{ \sqrt{1 + \frac{2K_s \tau_s}{\sqrt{\lambda_2}}} + K_s^2 \frac{\lambda_1}{\lambda_2} \right\} - 1
\]

Note that Eqs. (6) and (7) follow from Eqs. (12) and (13) by selecting \( \lambda = \lambda_2 \) and \( \lambda_1 = 0 \).

Due to the extra term in Eq. (13) the values of \( K_d \) which are found with criterion (9) are greater than with the more simple criterion (2). The values of \( K_p \) found from Eqs. (10) and (11) are in the order of magnitude of the values of \( K_p \) found by using Koyama's criterion.
3.3 Filtering of disturbances

The optimum controller gains which have been computed until now were all based on the assumption that the steering machine may be neglected. In other words, it was assumed that rudder angles of any amplitude could be realized without any delay. The block diagram of Figure 2 makes it clear that in practice the rudder amplitude as well as the rudder speed are limited. These two limitations prevent effective compensation of great disturbances of high-frequency.

The rudder limit limits the momentum which can be applied to counteract, for instance, the moments caused by the waves. On the other hand, the rudder limit is essential in order to prevent great phase lags which occur when large and fast rudder motions are ordered. Because this phase is caused by a non-linearity it cannot be compensated for by enlarging the rate feedback gain or by introducing other feedback signals. The only way to limit this phase lag is to limit the rudder angle itself. The maximum frequency of the desired rudder-angle signal which can be allowed is thus dependent on the rudder limit and on the rudder speed. With the aid of Figure 3 a formula for this maximum frequency can be derived.

Figure 3. The influence of the limited rudder speed
Suppose that $\delta_r$ is sinusoidal and that the amplitude is so large that after limiting $\delta_r$, the signal $\delta_r^1$ may be approximated by a block signal. The time needed for the rudder to move from $-\delta_{\text{max}}$ to $+\delta_{\text{max}}$ is then approximately

$$T_\delta = 2 \frac{\delta_{\text{max}}}{\dot{\delta}_{\text{max}}}$$  \hspace{1cm} (14)

When the non-linear steering machine is replaced by the linear first-order system

$$\frac{1}{\tau_\delta s + 1}$$  \hspace{1cm} (15)

the time constant $\tau_\delta$ may be very roughly approximated by

$$\tau_\delta \approx T_\delta = 2 \frac{\delta_{\text{max}}}{\dot{\delta}_{\text{max}}}$$  \hspace{1cm} (16)

However, this yields too large a value of $\tau_\delta$. A better approximation is found when the rudder-speed limiter is described by its describing function. This yields

$$\tau_\delta \approx \frac{\pi}{2} \frac{\delta_{\text{max}}}{\dot{\delta}_{\text{max}}}$$  \hspace{1cm} (17)

In order to allow the steering machine to be neglected, the time constant $\tau_\delta$ has to be 3-5 times smaller than the time constant $\tau_S$ of the Nomoto model (see Figure 1).

This yields

$$\frac{\pi}{2} \frac{\delta_{\text{max}}}{\dot{\delta}_{\text{max}}} < \frac{1}{3} \tau_S$$  \hspace{1cm} (18)

or, approximately

$$\frac{\delta_{\text{max}}}{\dot{\delta}_{\text{max}}} < 0.2 \tau_S$$  \hspace{1cm} (19)

For a small ship where $\tau_S \approx 10$ sec, and $\dot{\delta}_{\text{max}} \approx 5$ deg/sec this implies that $\delta_{\text{max}} \approx 10$ degrees. In general, this limitation will be
more severe on small ships than on large ships. This result gives not only a rule for the adjustment of $\delta_{\text{max}}$, but also gives an indication about the cut-off frequency of a filter which should suppress useless high-frequency rudder motions.

It can be seen from Fig. 3 that in the given situation, the steering machine introduces a substantial phase lag. This means that the rudder motions will enlarge the disturbance rather than reduce it.

The period of $\dot{\delta}$ which leads to this undesirable situation is

$$T = 2\pi \delta = \frac{4}{\delta_{\text{max}}}$$

This yields the frequency

$$f = 0.25 \frac{\dot{\delta}_{\text{max}}}{\delta_{\text{max}}}$$

In order to maintain a safety limit, this implies that frequencies greater than $f_{\text{max}} = 0.2f$ should be suppressed (at least when these frequency components have a great amplitude).

This yields the formula

$$f_{\text{max}} = 0.05 \frac{\dot{\delta}}{\delta_{\text{max}}}$$

In Table I, $f_{\text{max}}$ is given for several values of $\delta_{\text{max}}$ and $\dot{\delta}$.

<table>
<thead>
<tr>
<th>$\delta_{\text{max}}$</th>
<th>$\dot{\delta} = 2.5$</th>
<th>$\dot{\delta} = 3.5$</th>
<th>$\dot{\delta} = 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.025</td>
<td>0.035</td>
<td>0.050</td>
</tr>
<tr>
<td>10</td>
<td>0.013</td>
<td>0.018</td>
<td>0.025</td>
</tr>
<tr>
<td>15</td>
<td>0.008</td>
<td>0.012</td>
<td>0.017</td>
</tr>
<tr>
<td>20</td>
<td>0.006</td>
<td>0.009</td>
<td>0.013</td>
</tr>
</tbody>
</table>

Table I. $f_{\text{max}}$ as function of $\dot{\delta}$ and $\delta_{\text{max}}$
Of course these figures have to be carefully interpreted. The assumption made is that the desired rudder angles indeed exceed the rudder limit. This will depend on the amplitude of the disturbance signal and on the magnitude of the controller gains. Most problems may be expected when high controller gains are used for small ships sailing in rough weather. When the desired rudder angles are small and the rudder limit is not exceeded, the steering machine is not anymore the most limiting element. The low-pass character of the ship's transfer function will then determine whether motions of a certain frequency make sense or not. This is also related to wear and tear of the steering gear.

The following can be concluded:

- In order to allow the steering machine to be neglected and to allow Equations (6) and (7) or (12) and (13) to be used for controller adjustment the rudder limit should be chosen in accordance with inequality (19).

- To prevent rudder motions from enlarging the motions due to disturbances rather than suppressing them, filtering is essential. The need to filter depends on the amount of high-frequency disturbances. When this amount is small wear and tear of the steering gear can be prevented by suppressing frequencies larger than

$$f_{\text{max}} = \frac{1}{2\pi} \cdot \frac{1}{T_S}$$

(23)

because of the low-pass character of the ships transfer function.

The adaptive filter which was mentioned before is well suited to this purpose. The cut-off frequency of this filter is determined by the nature of the disturbances. This guarantees a maximum bandwidth of the system. Because of the predicting character of this filter, it does not introduce phase lag for variations in the state variables which are a result of rudder motions.
4. Experimental results

The ideas developed in the foregoing have been tested in various experiments. To judge the performance of different controllers the following approaches were used:
- simulation experiments
- model tests
- full-scale trials

Although specific difficulties are involved with each of the approaches all the results indicate the same optimum strategy.

Simulation experiments

By using an extensive mathematical model of the ship's dynamics the steering performance can be examined at the laboratory. The simplest method for obtaining a performance index which can always be used is to take one of the quadratic criteria discussed before. It is also possible to bring the dynamics which describe the loss of speed into the simulation.

Another approach is to use the observation made during the full-scale trials with a particular ship, the R.O.V. Zeefakkel, that the stationary loss of speed \( \Delta U \) is very well described by the relation (De Keizer, 1977):

\[
\Delta U = 0.013 |\delta| \cdot U
\]  

(24)

Because this relation contains no dynamics it will give an upper bound of the profits which may be expected by better steering. However, it should be noted that by using criterion (24) no penalty is put on the yawing motion itself, or on the elongation of the distance due to course errors.

The "loss of speed" due to these course errors can be added by extending Eq. (24) with the term

\[
\Delta U_2 = (1 - \cos \varepsilon) U
\]  

(25)

In Figure 4 results are given of simulation experiments (Van de Gaag, 1979) where the controller is optimally adjusted according to criterion (2)
with

\[ 0.3 < K_p < 3.2 \quad (0.1 < \lambda < 10) \]

The following controllers are compared:

a) without filtering
b) with adaptive filtering
c) with a dead zone of 2°
d) with a dead zone of 3°
e) with a dead zone of 4°

![Graphs showing measured loss of speed in simulation experiments](image)

Figure 4. Measured loss of speed in simulation experiments

The speed loss is measured in two different ways. In Fig. 4.a the loss of speed is measured with criterion (2) and \( \lambda = 10 \). In Fig. 4.b Eq. (26) is used:

\[ \Delta U = (1 - \cos \varepsilon)U + 0.013 \| \delta \| U \]  

(26)

During the experiments colored noise was added to the rudder angle to
simulate a sea state 4. The simulated ship is approximately described by the transfer function

$$\dot{\psi} = \frac{0.1}{10s+1}$$  \hspace{1cm} (27)

The length of this ship is 42 meters and the cruising speed 12 knots. From these figures it can be seen that without filtering low-controller gains yields a better economy, for both criteria. When a filter or a dead band is applied the adjustment of the controller gains is less critical; the improvement in the economy by this filter is considerable.

A dead band appears to be very effective with respect to the economy but it has disadvantages when more accurate steering is wanted. It should also be noted that in this idealized simulation no wind or other very low frequency disturbances were added, which enhances the results of the dead bands.

**Model tests**

A description is given in Van Amerongen et al. (1980) of a series of model tests which was carried out to measure the profits which could be obtained in different situations. The increased performance due to filtering is clearly measured. Depending on the situation profits from 0.3 to 5.6 per cent were measured. These measurements using a model of a ship of 150 meters length with a cruising speed of 13 knots, also seem to indicate that in rough weather low controller gains are more profitable, while in more smooth water high gains give the best performance (at least in combination with a filter).

**Full scale-trials**

Full-scale trials, reported by Van Amerongen et al. (1980), indicate savings of 0.5 - 1 per cent when an adaptive autopilot is used (with noise reduction filter and parameters adjusted according to Eq. (2) with $\lambda = 10$) in comparison to a conventional, well-adjusted autopilot.
5. Conclusions

The ideas of the foregoing sections can be summarized and transformed into a few rules for optimum autopilot adjustment.

During course changing the only setting which is required by the user is a setting to determine the stationary turning rate. The begin and end of the course-changing manoeuvre are completely fixed by the demand that the intention of the manoeuvre has to be clearly shown to other ships. The optimum course-changing manoeuvre can be defined by a step response with user-adjustable slope as given in Figure 2.

During course keeping optimum steering cannot be defined without taking into account the influence of the disturbances. It has been shown that it is essential to remove large high-frequency components from the desired rudder angle in order to prevent the rudder from enlarging the motions which should be reduced.

Maximum accuracy can be obtained by choosing the proportional gain, $K_p$, as large as possible while the rate feedback gain, $K_d$, is adjusted in order to guarantee sufficient damping. In practice, the value of $K_p$ is limited, approximately in the range

$$0.5 < K_p < 5$$

This controller setting could be achieved by using criterion (2) or (9) and selecting a value of $\lambda$ or $\lambda_2$, according to Equation (6) or (12). This yields for $K_p = 5$

$$\lambda = \lambda_2 = \frac{1}{K_p} = 0.04$$

(28)

In rough weather the high controller gains make good filtering even more essential than when low gains are used.

Maximum economy is mainly achieved by preventing the ship from turning.
As long as the rudder is able to prevent turning, this can be accomplished by using high controller gains based on low values of $\lambda$, which correspond to accurate steering.

It has been shown that this strategy is only effective when there are not too many high-frequency disturbances. When the weather is rough the rudder is not able to compensate for the high-frequency components of the disturbances and these must, therefore, be suppressed. Low values of $\lambda$, which heavily weight the course error in the criterion, are not useful with respect to the economy in this situation. Criterion (2), with $\lambda$ chosen according to Koyama, will then better define optimum steering.

Economical steering requires thus:

- In good weather: tight control with approximately the same controller settings as used for accurate steering.

- In bad weather: filtering and simultaneously lower controller gains, which also limit the bandwidth of the system.

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