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AN AUTOPILOT FOR SHIPS DESIGNED WITH FUZZY SETS

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An autopilot for ships designed with the aid of fuzzy sets is investigated. For this design the steering behaviour of a human controller is translated into a fuzzy mathematical model. The complexity of this model requires a digital computer for implementation of the autopilot.
The steering problem is split up into two regions with different control algorithms, a manoeuvring or course-changing mode and a course-keeping mode. Fuzzy sets are used also in the formulation of a fuzzy performance index that is based on human appreciation of the steering. The results are tested on a simulator, and a comparison is made between the fuzzy and a conventional PID controller which is most commonly used in autopilots.

1. INTRODUCTION

In general, control engineering methods are based on mathematics, in which precise information is required. Human operators follow a different approach to control a process; they do not rely on mathematical models, etc., but mostly they use only vague information. Is it not surprising that many complex processes can be controlled very well by human operators? In some cases their performance is excellent and better than that of an automatic control device.

Especially in non-engineering systems the available information is more or less vague and subjective, and the application of ordinary control methods can hardly be expected to lead to success.
The theory of fuzzy sets introduced by Zadeh[1] gives the possibility of taking into account vague and subjective information and of using this information in the design of a controller. Although simple examples [2] [3] of this approach are reported in the literature, this paper describes a fairly complicated controller, an autopilot for ships. In this application attention is given to certain disturbances in the ship steering behaviour due to the state of the wind and sea.

Besides, the judgement of the performance of the controller is an interesting point. Like in many other systems, the formulation of an integral criterion or something similar does not correspond with the human appreciation of the system. In the autopilot system an attempt is made to define a fuzzy performance index that is in better agreement with the goals of a human controller.

There were several reasons for selecting this particular system, an autopilot for ships, as a test case.
- It may be that fuzzy sets are especially suitable for non-engineering systems; however, applying a fuzzy controller to a well-known technical system gives the opportunity to compare its results with those of an ordinary non-fuzzy controller.

- Although many ships are equipped with an autopilot today, the steering is taken over by a helmsman in more difficult and dangerous situations. Apparently ship steering is not as simple as it may seem at first sight.

- At the control engineering laboratory a lot of research has been done in the field of ship steering. Investigations into the design of an adaptive autopilot especially gave valuable information [4].

This research and field tests have led to many ideas about formulating a "good steering behaviour."

2. GENERAL DESIGN CONSIDERATIONS

In control engineering the starting point usually is the system to be controlled. From the model of the system the controller is designed. In the method applied to design a fuzzy controller, there is a diversion of the attention from the system to be controlled to the human being controlling the system satisfactorily. Basically, the approach for the general design of the controller comprises two steps.

The first step is reached by observing and interrogating the human controller. It is not possible to point out a generally applicable, systematic approach to this "information extraction" process.
It is evident that the procedure will be heuristic and subjective.
The information gathered this way is to be "processed", until a summary is obtained of all control rules in an ordered sequence of linguistic conditional statements ("linguistic description").

Let the human controller receive the information from m sources:

$u_1, u_2, \ldots\ldots u_m.$

And let him control n variables:

$y_1, y_2, \ldots\ldots y_n$.

The general form of a linguistic conditional statement can then be formulated:

If u_1 is A_1 and ....and u_m is A_m,

then y_1 is B_1 and...and y_n is B_n.

$A_1,\ldots.A_m$ and $B_1,\ldots\ldots B_n$ are linguistic qualifications of the source and control variables, like small, big, very small, fast, not so fast, etc.

The second step is to transform the linguistic description of the control function into a mathematical model. This is done with fuzzy set theory. The vague expressions occuring in the linguistic description are translated into fuzzy sets and the linguistic conditional statements, into fuzzy relations between these fuzzy sets. The resulting fuzzy algorithms are then manageable for a digital computer.
In fact this step is no more than choosing suitable membership functions for all the fuzzy notions that occur in the linguistic description.
Also for step 2 there do not exist directives: within certain boundaries, the number of possibilities is unlimited.
Due to the (fuzzy) methods applied, the mathematical model of the human control function will be non-linear. It should be noted that in general any further implementation without a digital computer would be impossible.

In this investigation the theory has been applied to the ship steering problem. In this context automatic steering of a ship is defined as replacing the helmsman by an automatic controller. In other words, the desired heading is set by the officer of the watch, the controller then takes care that this heading is reached or maintained as well as possible. The ship under consideration is the naval training vessel "Zeefakkel" (length 45 m, largest width 7,5 m, displacement 392 ton, draught 2,2 m, power 640 HP, two propellers with variable pitch).
For simulation a simplified non-linear transfer function of this ship is available. It gives a relation between the commanded rudder angle and the rate of turn of the ship for constant thrustpower.

In observing the behaviour of a human controller, it turns out that significant differences exist in the course-changing procedure and the course-keeping procedure: in the following referred to as far-away control mode and close-by control mode, respectively. Therefore, these two modes are treated separately.

3. FAR-AWAY CONTROL

During course-changing the task of the helmsman is to reduce the course error quickly, without too much overshoot and with a not-too-large rate of turn.
He uses the following information:

a. commanded heading ψ_c, according to instructions of the officer of the watch.

b. true heading ψ_r, given by the compass.

c. rate of turn $\dot{\psi}_r$. Usually this information is not directly available, but it is derived from the compass or from observing fixed shore marks and clouds.

With this information the helmsman has to control the rudder angle δ_c.
In analogy it has been decided that the fuzzy controller would have the same two inputs $\varepsilon = (\psi_c - \psi_r)$ and $\dot{\psi}_r$. And the output δ_c (Fig. 1).

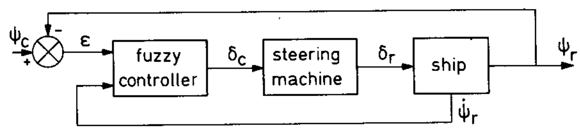

Fig. 1

Usually a course-changing manoeuvre is carried out in three steps: 1. rudder; 2. zero; 3. counter-rudder (Fig. 2).

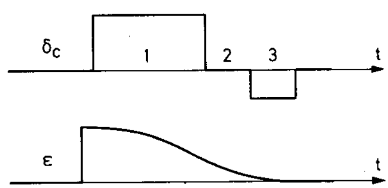

Fig. 2

The input information can be presented in the ε-$\dot{\psi}_r$-plane. This plane can be divided into fuzzy areas in such a way that in each area a certain action is dominant. (see Fig. 3).
Note that the ε-$\dot{\psi}_r$-plane is the reversed phaseplane. Trajectories go counterclockwise as is shown with one example in the figure.
If fuzzy sets are defined on ε and $\dot{\psi}_r$, then the ε-$\dot{\psi}_r$-plane is divided into "fuzzy rectangles", with which the areas as given in Fig. 3 may be approximated.
In order to keep the controller as simple and straightforward as possible, the number of fuzzy sets on ε, $\dot{\psi}_r$ and δ_c is, to begin with, restricted to five.

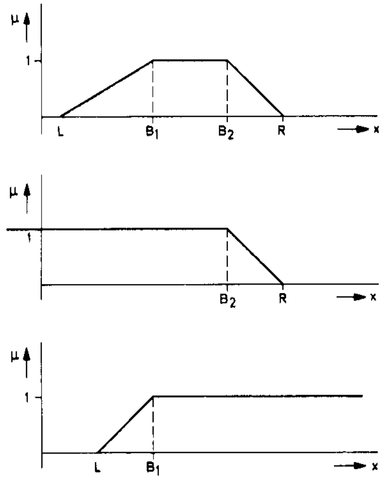

Fig. 4

These fuzzy sets are labelled:
- negative big (NB)
- negative small (NS)
- approximately zero (Z)
- positive small (PS)
- positive big (PB).

Again, for reasons of simplicity all fuzzy sets are standardized to one of the forms in Fig. 4. Each fuzzy set has adjustable parameters: B_1 (breakpoint 1), B_2(break point 2), L (left), R(right).

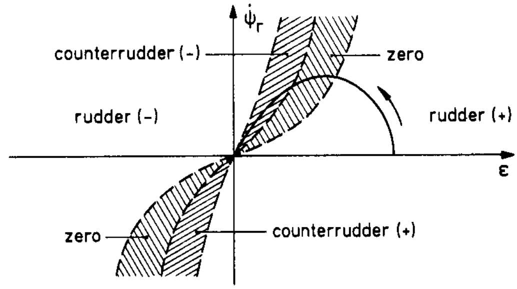

Fig. 3

With these simplifications the membership functions (μ) of these 15 fuzzy sets are shown in Fig. 5.

The following framework is chosen as being a good approximation of the areas of Fig. 3:

$\dot{\psi}_r$ \ ε (δ)	NB	NS	Z	PS	PB
PB	NB	NB	NB	Z	PB
PS	NB	NB	NS	PS	PB
Z	NB	NS	*	PS	PB
NS	NB	NS	PS	PB	PB
NB	NB	Z	PB	PB	PB

* = close-by control

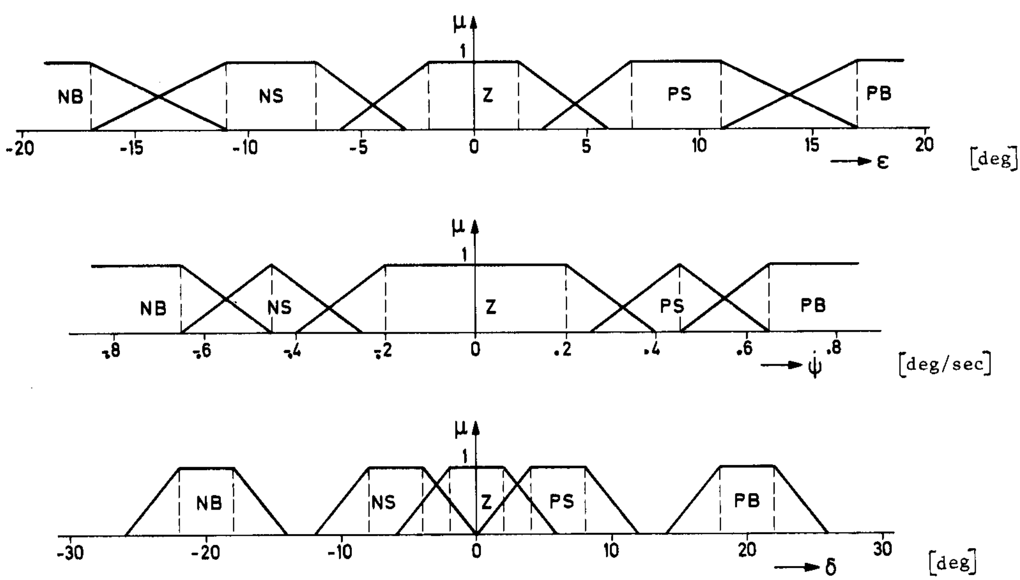

Fig. 5

The far-away algorithm can be summarized in 12 statements as follows:

1. If ε is PB and $\dot{\psi}_r$ is any , then δ_c is PB.
2. If ε is PS and $\dot{\psi}_r$ is NS or NB , then δ_c is PB.
3. If ε is PS and $\dot{\psi}_r$ is PS or Z , then δ_c is PS.
4. If ε is PS and $\dot{\psi}_r$ is PB , then δ_c is Z .
5. If ε is Z and $\dot{\psi}_r$ is NB , then δ_c is PB.
6. If ε is Z and $\dot{\psi}_r$ is NS , then δ_c is PS.
7. If ε is Z and $\dot{\psi}_r$ is PS , then δ_c is NS.
8. If ε is Z and $\dot{\psi}_r$ is PB , then δ_c is NB.
9. If ε is NS and $\dot{\psi}_r$ is NB , then δ_c is Z .
10. If ε is NS and $\dot{\psi}_r$ is NS or Z , then δ_c is NS.
11. If ε is NS and $\dot{\psi}_r$ is PS or PB , then δ_c is NB.
12. If ε is NB and $\dot{\psi}_r$ is any , then δ_c is NB.

As a definition for the conditional statement, the cartesian product is chosen [2,5]. This means that the output fuzzy set is defined by

$$\mu_o(\delta_c) = \max_{n=1,\ldots,12}$$

$$\left[\min\{\mu_{\varepsilon_n}(\varepsilon), \max(\mu_{\dot{\psi}_{n_1}}(\dot{\psi}_r), \mu_{\dot{\psi}_{n_2}}(\dot{\psi}_r)), \mu_{\delta_{c_n}}(\delta_c)\}\right]$$

During calculation in the computer, δ_c was quantified at 25 levels:

-30^o, -27.5^o, ..., 30^o .

In order to choose a deterministic value, $\delta_{c,act}$, the center-of-gravity decision procedure is applied to the output fuzzy set [3]:

$$\delta_{c,act} = \frac{\sum_{d_c=-30}^{30} \delta_c \mu_o(\delta_c)}{\sum_{d_c=-30}^{30} \mu_o(\delta_c)}$$

Evidently, $\delta_{c,act}$ can take any value between -30^o and 30^o.

The algorithm is assumed to be fixed; the parameters of the 15 fuzzy sets are adjustable. Subjectively, and based on observations of the helmsman, an initial set of membership functions has been chosen. The idea is to apply modifications in a more or less systematic way in order to observe their influence on the performance. The performance of the fuzzy controller tested in a simulation of the system is judged subjectively.
The initial set of membership functions has already given good results. Several small changes in the parameters of the membership functions have been tried out to optimize the performance. It has turned out that only slight improvements are to be obtained. The precise shape of the fuzzy sets is not critical in this example. In practically all cases the fuzzy controller can perform any imposed change in the course satisfactorily, i.e., in a reasonable time, with a reasonable rate of turn, and without too much overshoot in course. The finally selected numerical values of the membership functions are given in Fig. 5.

4. CLOSE-BY CONTROL

Course-keeping requires a special technique and is rather difficult. High frequency components in the course must be neglected, because these cannot be compensated for by rudder movements. Besides, too many rudder motions make the steering machine wear away quickly. A moving average of the course error must be detected for correction of the mean heading of the ship. Most of the correcting is done in "rudder-gusts". This means a certain rudder angle is set and, before the ship begins to move, brought back to the initial value again.

For close-by control one statement is added to the 12 statements of the far-away control algorithm:

13. If ε is Z and $\dot{\psi}_r$ is Z, then δ_c is computed by the close-by mode.

An automatic close-by controller should be reasonably accurate with not too many rudder calls per unit of time and not too great a loss of speed of the ship due to rudder motions.
It is not easy to design the close-by mode of the fuzzy controller. A system that combines some features of ordinary and fuzzy control principles has given the best results.
To make stationary errors small, a conventional integrating action is placed parallel to the fuzzy controller:

$$\delta_{perm.\ helm} = K_i \int_0^t \varepsilon \, d\tau .$$

K_i may have a small value. The I-action is switched off when ε exceeds a certain value.

For faster corrections, rudder signals are given "in gusts" as shown in Fig. 6: pulses with amplitude D and width T_1, not to be repeated within time T_2.

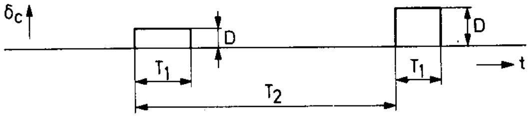

Fig. 6

D is a fuzzy variable, which can take the following values:

negative small (NS)
negative very small (NVS)
negative very very small (NVVS)
approximately zero (Z)
positive very very small (PVVS)
positive very small (PVS)
positive small (PS).

The corresponding membership functions are drawn in Fig. 7.

The pulse width T_1 has a minimum value T1MIN and a maximum value TIMAX1.
The computation of D and T_1 is based on the following observations of the helmsman.
If ε is very small, but not zero, during a larger period of time a correction is made by computing a rudder-gust, with amplitude D and duration T_1, dependent on the mean value of ε.
If ε is not very small, a faster correction is required, but it is still based upon the mean value of ε.
This policy inherently "filters" a noisy ε signal and does not use a dead zone which is usually applied in autopilots.

No rudder is given until the following inequality is valid:

$$\left| \int_0^{T_2} \varepsilon \, dt \right| > S \quad .$$

The integration is reset any time a rudder-gust is given and also when ε crosses 0.
Then the average error is computed as

$$\varepsilon_{av} = \frac{1}{T_2} \int_0^{T_2} \varepsilon \, dt \ ,$$

and the required amount of rudder is calculated.

A variable C is introduced:

$$C = K_p \varepsilon_{av} + K_d \frac{\varepsilon_{av} - \varepsilon_{avl}}{T_s}$$

(ε_{avl} = foregoing average error,
T_s = sample time).

T_1 and D are determined as follows.

Let T = T1MIN + abs (C).

Then:

1. If C < 0 and T<TIMAX , then D is NVVS during T_1 = T.
2. If C > 0 and T<TIMAX , then D is PVVS during T_1 = T.
3. If C < 0 and TIMAX<T<TIMAX2, then T1 = TIMAX and D is NVS during T_1 = T1MAX.
4. If C > 0 and TIMAX<T<TIMAX2, then T1 = TIMAX and D is PVS during T_1 = T1MAX.
5. If C < 0 and T>TIMAX2, then D is NS during T1 = T1MAX.
6. If C > 0 and T>TIMAX2, then D is PS during T1 = T1MAX.
7. Between the pulses, D is Z.

Without changing the fuzzy sets in principle, the parameters of the controller K_i, K_p, K_d, T1MIN, T1MAX and T1MAX2 are adjusted to get the best results.
However, when disturbances, or noise to simulate a rough sea state, are applied to the system, the controller runs out of its "close-by area" quickly. Therefore, the orginal algorithm is modified. Statements 6 and 7 are omitted. Statement 13 is changed into:

If ε is Z and $\dot{\psi}_r$ is any then δ_n is D.

This change improves the close-by behaviour considerably.

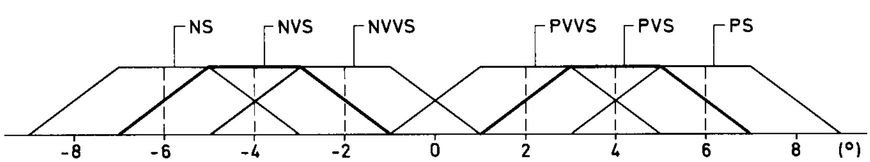

Fig. 7

5. FUZZY PERFORMANCE INDEX

The performance of an automatic controller may be judged from several viewpoints. In the case of ship steering, for instance, speed, economy, safety, comfort and accuracy are more or less important aspects. How they should be weighted depends upon the circumstances. A human being will judge each of these aspects in a "fuzzy" way: in terms of "good", "reasonable", etc. Also he will attach certain vague grades of importance, weighting factors, to each aspect: like "very important", "not so important", etc. This will enable him to formulate in fuzzy terms a final judgement of the performance of the controller as a whole.
With the aid of fuzzy set theory the same procedure can be carried out by a digital computer.

As an experiment, an attempt has been made to measure "fuzzily" the performance of autopilots this way.

The aim is to qualify an autopilot with a mark between 1 and 10 (1 = very bad, 10 = excellent). Aspects taken into account are response time, loss of speed, rate of turn, number of rudder calls, and accuracy.
A standard test program has been drawn up on which the judgement is based.
The test program involves several turns of different sizes as well as a stretch of course-keeping, and lasts about 6 minutes.

Fuzzy sets are assigned as follows:

1. The response time during course-changing. The time between the moment a heading is set and the moment the ship is on the new course. Since, evidently, Δt is dependent on the size of the change in course ($\Delta\psi$), $\frac{\Delta t}{|\Delta\psi|}$ is chosen to be the fuzzy variable. Three fuzzy sets are defined, called "slow", "normal" and "fast".

2. The loss of speed during course-changing and course-keeping. According to [6] $\int \delta^2 \, dt$ is used as a measure for the speed loss. This aspect is important from an economical point of view. Because the integral depends upon $\Delta\psi$, $\int \delta^2 \, dt / |\Delta\psi|$ is chosen to be the fuzzy variable. Two fuzzy sets are defined "normal" and "big".

3. The rate of turn during course-changing. A too large rate of turn may cause inconvenient or even dangerous centrifugal forces on the ship and its crew and is, therefore, unfavourable. The fuzzy variable is the maximum value of $\dot{\psi}_r$ in degrees/sec during the manoeuvres. The fuzzy sets are "normal" and "big".

4. The number of rudder calls per unit of time during course-keeping. The less rudder calls the better, in view of steering machine wastage. The fuzzy variable is the number of rudder calls in 50 seconds. The fuzzy sets are "few", "normal", "many".

5. The accuracy during course-keeping. The fuzzy variable is

$$\varepsilon_{av} = \frac{1}{50}\int_0^{50} |\varepsilon| \, dt \quad \text{(in degrees)}.$$

The fuzzy sets are "small", "normal", "high".

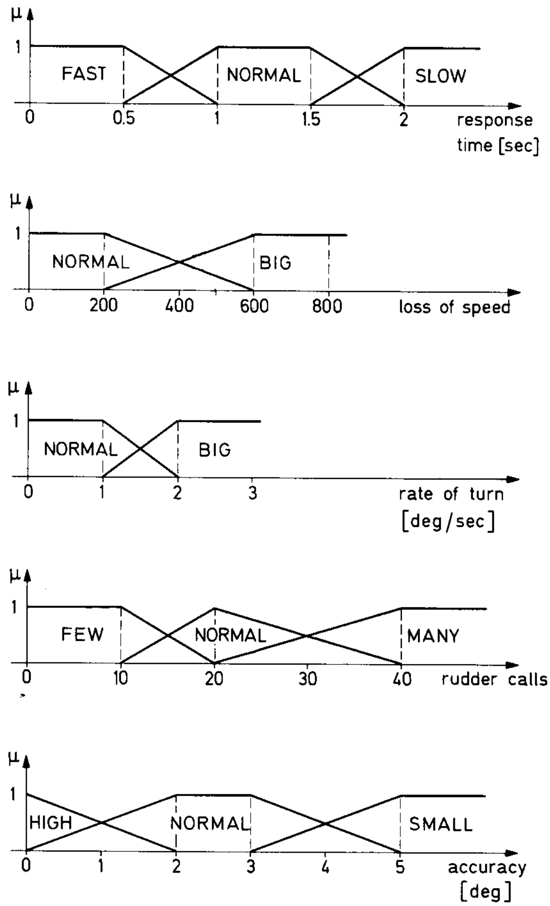

Fig. 8

In Fig. 8 the fuzzy sets are given.

With weighting factors between 0 and 1, non-fuzzy in this application, "grades of importance" are assigned to each of the five aspects.

Each of the membership values of an aspect, calculated on the basis of the test program, is multiplied by the weighting factor corresponding to the particular aspect. These "modified" membership values are then fed into the following algorithm:
1. If the response time is fast, loss of speed is normal, rate-of-turn is normal, number of rudder calls is few, and the average-course error is small, then the controller is excellent, denoted by a mark 10.

2. If the response time is normal, number of rudder calls is normal, average-course error is normal, loss of speed is big and rate-of-turn is big, then the controller is reasonable, denoted by a mark 5.5.

3. If the response time is slow, loss of speed is big, rate-of-turn is big, number of rudder calls is many and average-course error is big, then the controller is very bad, denoted by a mark 1.

Intermediate performances get a mark between 1 and 10.

The performance algorithm is calculated with the same definition (the cartesian product) for a conditional statement and the same decision procedure (the center of gravity) as the control algorithm.

6. RESULTS AND CONCLUSIONS

The algorithms of the former sections have been implemented in the PDP-11/20 computer of the control laboratory. Via AD- and DA-converters the computer is connected to an analogue simulation of the ship.
A non-linear, fourth-order differential equation describes the ship's dynamics. Coloured noise, with a spectrum between 0.1 Hz and 0.5Hz can be added to the rate of turn signal to simulate random disturbances (sea-state). A constant disturbance signal simulates the influence of the wind.

A test run is defined consisting of the following course-changing manoeuvres:

1. $\Delta\psi = 10^o$
2. $\Delta\psi = -20^o$
3. $\Delta\psi = +30^o$
4. $\Delta\psi = -30^o$
5. $\Delta\psi = +20^o$
6. $\Delta\psi = -10^o$
7. $\Delta\psi = 0^o$.

The total run time is 7 x 50 sec. = 350 sec.

During the course-changing manoeuvres (1-6), the following criteria are computed:
response time, loss of speed and rate of turn.
During course-keeping (7) loss of speed, number of rudder calls, and accuracy are computed.
Finally marks between 1 and 10 are computed for each of these aspects and a total mark, for the controller as a whole.
To compute this final mark all aspects are weighted equally with the exception of the accuracy which is weighted with a factor of 0.5.
The fuzzy sets of the controller are given in Fig. 5. The fuzzy sets of the performance are given in Fig. 8.

To compare the results of fuzzy control and conventional control, a PID-controller has been also implemented in the digital computer. The integrating action is similar to the integrating action in the fuzzy controller. The differentiating action is realized as a rate feedback. K_p, K_d and K_i denote the gains of proportional, differentiating and integrating action, respectively. The PID-controller is further extended, with a linear, second-order, low-pass filter with a transfer function

$$H_f = \frac{1}{s^2 + s + 1}$$

Then the rudder angle is computed as

$$\delta_c = H_f.\{K_p\varepsilon - K_d\dot{\psi}\} + K_i \int_0^t \varepsilon dt .$$

The filter prevents rudder motion from occurring at too high a frequency and does not influence the dynamic behaviour of the system.

In Fig. 9 and 10 some typical responses are given of a fuzzy controller and of a PID-controller, respectively. A course-change of 30^o is followed by a period of course-keeping. A constant disturbance and noise are added. The PID-controller settings are

$$K_p = 1,\ K_d = 5,\ K_i = 0.2.$$

The fuzzy controller settings are

$$K_p = 2,\ K_d = 1,\ K_i = 0.2,\ TIMAX = 2.5\ ,$$

$$TIMAX2 = 5,\ S = 10,\ TIMIN = 2.$$

The fuzzy controller clearly shows about the same behaviour as that of a helmsman. During course-changing the phases of rudder, zero-rudder and counter-rudder can be recognized, and during course-keeping, rudder-gusts are given.

In the design of the fuzzy controller special attention has been paid to the prevention of many rudder motions.
The result can be seen in Fig. 9 and Fig. 10, but even more clearly in the table of Fig. 11 where the performance of the PID- and fuzzy-controller is compared for various sets of parameters, both with and without noise.

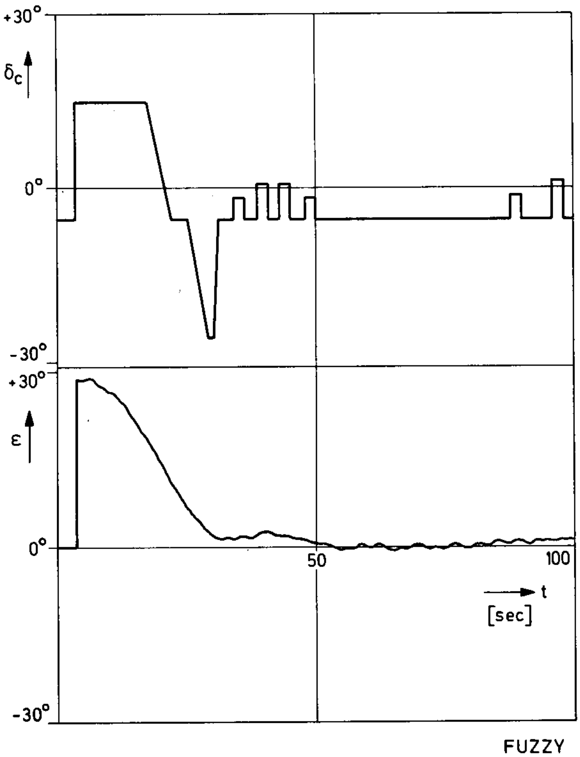

Fig. 9

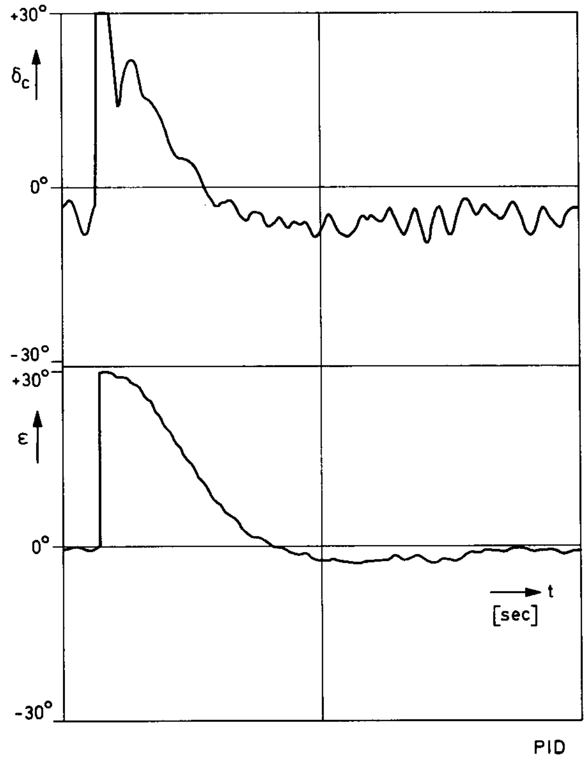

Fig. 10

When noise is added the given marks are mean values of at least three runs.

The following conclusion can be drawn:

- In the noise-free case, the PID- and fuzzy-controller get, if optimally adjusted, about the same marks (column 3 and 5-9); however the fuzzy controller is less sensitive to variations in the control parameters.
- When noise is added the fuzzy controller performs significantly better than the PID-controller, especially concerning the number of rudder calls.
  Besides, the same settings of the fuzzy controller can be maintained, while the settings of the PID-controller have to be changed dependent on the noise level, to keep the performance optimum.
- The fuzzy controller is much more difficult to design than the PID-controller. Once suitable fuzzy sets have been chosen, the adjustment procedure is relatively insensitive to parameter variations.
- The fuzzy performance index gives more insight into the relation between the desired performance and what is realized, than a conventional integral criterion. Subjective judgements of the performance can easily be expressed in the criterion.

In Fig. 11 the following abbreviations are used:

t_r = response time

$\int \delta^2$ = loss of speed

$\dot{\psi}$ = rate of turn

N_δ = number of rudder calls

acc= accuracy

TYPE:	PID	PID	PID	PID	FUZ	FUZ	FUZ	FUZ	FUZ	PID	PID	PID	FUZ	FUZ	FUZ	FUZ	FUZ
K_1	.5	1	2	4	2	2	2	2	2	2	1	.5	2	2	2	2	2
K_2	2.5	5	10	20	10	5	2.5	1	1	10	5	2.5	1	5	5	1.0	1.0
K_3	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2	.2
TIMAX					5	5	5	5	2.5				2.5	5	2.5	2.5	5
TIMAX2					10	10	10	10	5				5	10	5	5	10
S					10	10	10	10	10				10	10	10	10	10
TIMIN					2	2	2	2	2				2	2	2	2	2
NOISE	-	-	-	-	-	-	-	-	-	+	+	+	+	+	+	+	+
t_r	1.0	4.1	5.5	5.2	5.1	5.8	3.8	4.4	4.8	5.5	4.6	1.0	3.1	2.5	2.8	1.9	2.4
$\int\delta^2$	10.0	5.9	5.5	5.5	10.1	9.8	10.0	10.0	10.0	5.5	5.5	10.0	6.5	5.5	5.5	5.5	6.4
$\dot{\psi}$	10.0	10.0	8.8	7.2	10.0	10.0	10.0	9.9	9.9	7.2	9.0	10.0	8.8	8.9	9.0	9.1	9.2
N_δ	10.0	9.1	5.5	1.0	7.8	9.4	8.7	9.8	10.0	1.0	1.1	4.1	9.5	8.6	10.0	7.3	6.4
acc	6.4	8.6	9.9	9.7	5.5	7.1	7.1	7.4	8.2	8.3	9.1	8.7	8.3	7.7	8.7	7.4	6.8
TOT	5.5	6.9	7.4	4.5	7.5	7.6	6.9	7.2	7.4	4.4	5.1	5.5	6.2	5.7	6.2	5.6	5.8

Fig. 11

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