Model Reference Adaptive Autopilots for Ships*

Modèle de Référence d’Autopilote Adaptable pour Bateaux

Adaptive Autopiloten mit Bezugsmodell für Schiffe

J. VAN AMERONGEN† and A. J. UDINK TEN CATE†‡

Variations in the steering characteristics of a ship necessitate adjustment of the autopilot settings, and model reference adaptive control systems perform the adjustment automatically.

Summary—For control purposes the steering characteristics of a ship at constant thrustpower can be described by a relatively simple mathematical model. The dynamic behaviour of the ship and hence also the parameters of this model are dependent on the external circumstances and the applied thrustpower. When the ship is steered with an autopilot it is necessary to adjust the parameters of the autopilot dependent on the change of the steering characteristics of the ship. The easiest way to do this is to adapt automatically the parameters of the autopilot. In this paper two methods of adaptation are compared. First a description is given of an autopilot, synthesized according to the ‘sensitivity model’ approach, especially designed to prevent the course instability which can occur for very large ships. Second a method of adaptation known as the stability (Liapunov) approach is used. By simulation with the model of a ship of 2000 brt, a comparison is made between the two methods. The results were tested in practice on this ship. During the measurements at sea special attention was paid to the problem of filtering the disturbances due to yawing.

1. INTRODUCTION

In the past several autopilots for ships have been developed. Generally, they are designed for keeping a constant course; some of them can also be used for a variable course. They are composed of rather simple controllers to correct the disturbances of the set course, and should also give the desired response at course changes. This can be achieved by adjusting the parameters of the controller in accordance with the steering characteristics of the ship. For small variations in the steering characteristics the feedback system corrects automatically the overall behaviour. For large variations the parameter settings of the autopilot must be adapted manually.

Especially for supertankers the variations can be large and they can appear suddenly, for instance when the depth of water changes. Due to their size these ships are also manually difficult to handle. Therefore, autopilots that can be used to make accurate course changes in narrow coastal waters, will be of great importance. Commonly applied autopilots have many parameter settings but they have to be set manually. The introduction of an automatically adaptive autopilot permits the application of a course feedback system in those situations where varying conditions give rise to difficulties until now.

It is known that a variation of depth of water may even introduce course instability. By application of an appropriate rate feedback this phenomena can be avoided. The gain of the rate feedback should be adapted automatically to obtain the desired behaviour of the system in all circumstances. In this paper two alternative methods are described and compared.

2. THE MATHEMATICAL MODEL OF A SHIP

Generally a ship can be described as a system with two inputs, rudder-angle and thrustpower, and two outputs, course-angle and speed. However, for the purpose of designing an autopilot such a model is too complicated. Most of the autopilots which are on the market at present are specially designed for keeping a fixed course with a course-stable ship. As in this case only small rudder-angles occur, a simple linear transfer function between the rudder-angle and the course can be used to describe the behaviour of the ship. This transfer function can be derived from the complicated, non-linear models that are used in shipbuilding technology. A simple
model is described by Nomoto [1]:

\[ \tau_1 \tau_2 \dot{\psi} + (\tau_1 + \tau_2) \ddot{\psi} + \psi = K\delta + K_3 \delta. \tag{1} \]

For the definition of symbols see Fig. 1. Using the Laplace transformation, (1) can be written

\[ \frac{\psi(s)}{\delta(s)} = \frac{K(s\tau_3 + 1)}{(s\tau_1 + 1)(s\tau_2 + 1)s}. \tag{2} \]

K, \tau_1, \tau_2 and \tau_3 are parameters related to the hydrodynamic coefficients, the mass and the speed of the ship.

![Diagram](image)

0 = ships center of gravity
\( \delta_w \) = rudder angle
\( \delta_g \) = desired rudder angle, input to steering gear
\( \psi_w \) = course
\( \theta_g \) = desired course
\( \dot{\psi}_w \) = course angular velocity

Fig. 1. Definition of symbols.

The rudder-angle should not exceed 5°. For our purpose this model is too simple. We also want to describe course-unstable ships and course-stable ships, using rudder-angles larger than 5°. Therefore, the transfer function is extended using a non-linearity as proposed by Bech [2].

When the thrustpower is assumed to be constant during the manoeuvres, then \( K/\tau_1 \tau_2, (\tau_1 + \tau_2)/\tau_1 \tau_2 \) and \( \tau_3 \) are approximately constant, whereas \( 1/\tau_1 \tau_2 \) changes considerably. Dividing (1) by \( \tau_1 \tau_2 \) and substituting (1/K) \( \psi = H(\psi) \), yields

\[ \ddot{\psi} + \left( \frac{1}{\tau_1} + \frac{1}{\tau_2} \right) \dot{\psi} + \frac{K}{\tau_1 \tau_2} H(\psi) = \frac{K}{\tau_1 \tau_2} (3\delta + \delta). \tag{3} \]

If the rudder-angular velocity is neglected [3] and if

\[ a_1 = 1/\tau_1 + 1/\tau_2 \quad \text{and} \quad K' = K/\tau_1 \tau_2 \]

equation (3) simplifies to

\[ \ddot{\psi} + a_1 \dot{\psi} + K' H(\psi) = K' \delta. \tag{4} \]

\( H(\psi) \) is a non-linear function of \( \psi \), and the other coefficients in (4) are constant if the external conditions do not change.

The function \( H(\psi) \) can be found from the relationship between \( \delta \) and \( \psi \) in the steady state (\( \ddot{\psi} = \dot{\psi} = \delta = 0 \)). Experiments, known as the spiral test, give this relationship; and for course-unstable ships the 'reversed spiral test' is used. \( H(\psi) \) can be approximated by

\[ H(\psi) = a_1 \psi^3 + b_1 \psi, \tag{5} \]

where the parameter \( a \) is always positive in value, but \( b \) can have positive and negative values. When \( b \) is negative the ship is course-unstable. \( H(\psi) \) is called the reversed spiral curve. The coefficients \( K' \) and \( a_1 \) in (4) can be found from zig-zag trials.

### 3. ADAPTATION

Due to changing conditions, such as loading, speed and depth of water, the parameters of (4) and (5) are variable. The loading of the ship influences \( H(\psi) \), \( K' \) and \( a_1 \). The depth of water influences primarily the function \( H(\psi) \), i.e. the shape of the reversed spiral curve.

In a certain range of depth of water, ships with a poor course stability can even become course unstable. The variations in \( K' \) and \( a_1 \) are much smaller. Details are given in [4].

Large parameter variations have to be compensated by means of adaptation. Variations in \( b \) are large compared with variations in the other parameters. When \( b \) becomes negative and the ship is steered with an autopilot with fixed settings, the ship will behave in a course unstable fashion.

The basic scheme of most autopilots is a PID-controller. For course-unstable ships, the differentiating action is realized as a rate feedback. The compensation of the course instability by an appropriate rate feedback can be easily shown. When the steering gear dynamics are neglected, which is permissible since they are fast in comparison with the ship dynamics, the rate feedback signal is added to the non-linearity:

\[ H'(\psi) = a_1 \psi^3 + b_1 \psi + K_d \dot{\psi}. \tag{6} \]

A decreasing parameter \( b \) can be compensated by an increasing parameter \( K_d \), the rate feedback gain. However, the variations in the ship's parameters are often unexpected and unpredictable. Therefore, manual adjustment of the autopilot settings is not practical and automatic adaptation is more convenient.

In Fig. 2 the block diagram of an adaptive autopilot, using a reference model, is given. In this configuration the model represents the desired behaviour of the ship. Two adaptive loops have been investigated and compared, one designed using a sensitivity model and the other according to the Liapunov approach.

In the adaptive system the transfer between \( \delta_g \) and \( \psi_w \) is considered.
The following approximations are made
(a) The function $H(\delta)\!$ can be approximated by a
third-order polynomial $a\delta^3 + b\delta$; the even
factors of this polynomial can be neglected;
for course-stable ships $a$ and $b$ are positive,
for course-unstable ships $a$ is positive and $b$
is negative.
(b) The coefficient $a$ of this polynomial is
assumed to be constant and known, whereas
the coefficient $b$ is varying.
(c) The parameters $K'$ and $a_1$ of the ship are
constant and known.
(d) The influence of the dynamics of the steering
gear on the behaviour of the system can be
neglected.
Approximation (b) is not necessary, but if it is
fulfilled the value of $K_d$ is independent of $\delta_p$. The
differential equations of the reference model and
the system are respectively
model:
\[ K_m' \delta_p = \ddot{\psi}_m + a_2m \dot{\psi}_m + K_m'(a_m \psi_m^3 + b_m \psi_m), \]  
(7)
system:
\[ K_w' \delta_p = \ddot{\psi}_w + a_2w \dot{\psi}_w + K_w'(a_w \psi_w^3 + b_w \psi_w), \]  
(8)
the error:
\[ e = \psi_m - \psi_w. \]  
(9)
\[ H(\delta) = a_w \psi_w^3 + b_w \psi_w, \]  
(10)
\[ e = b' - b_m, \]  
(11)
\[ b' = b_w + K_d. \]  
(12)
In these equations
\[ a_m = a_w, \]
\[ b_m = b_w, \]
\[ K_m' = K_w', \]
\[ a_1m = a_1w. \]
\[ K_d\] is the amount of rate feedback necessary to
obtain a satisfactory damped behaviour of the
course control system of the course-stable ship in
deep water. It is assumed that variations in $\delta$ due
to the adaptation are much faster than variations in
$\delta$ due to changes in the input signal of the ship
and the model.
Also $b_w$ must be varying slowly compared with the
rate of adaptation.

4. THE SENSITIVITY MODEL

Adaptation with a sensitivity model is in fact an
implementation of a continuous hill climbing
technique. The criterion that has to be minimized
is the difference between the responses of ship and
reference model. The criterion chosen is
\[ C = \int_0^\infty \frac{1}{2} \epsilon^2 \, dt. \]  
(13)
Because the error approaches zero after some time
interval $T$, (13) may be rewritten
\[ C = \int_0^T \frac{1}{2} \epsilon^2 \, dt. \]  
(14)

The adjustment of $K_d$ is made using a steepest
descent technique.
From (11) it follows that $\epsilon$ can be minimized by
adjusting $K_d$. According to the gradient method,
incremental changes in $\epsilon$ are made in the time
interval $\Delta t$,
\[ \Delta \epsilon = -K(\delta C/\delta \epsilon). \]  
(15)
For an infinitesimal period of time this yields
\[ \lim_{\Delta t \to 0} \frac{\Delta \epsilon}{\Delta t} = \frac{d \epsilon}{dt} = \frac{d}{dt} \left( -K \frac{\partial C}{\partial \epsilon} \right), \]  
(16)
\[ \dot{\epsilon} = \frac{\partial}{\partial \epsilon} \left[ -K \frac{dC}{dt} \right] = \frac{\partial}{\partial \epsilon} \left( -K(1/\epsilon^2) \right), \]  
(17)
\( \varepsilon \) can only be varied by varying \( b' \), thus

\[
\dot{\varepsilon} = \frac{\partial}{\partial b'} \{-K(\frac{1}{b} \varepsilon^2)\}.
\]

(18)

Calculation of (18) requires prior knowledge about \( b' \), which is not available. However, for small values of \( \varepsilon \), (18) can be approximated by

\[
\dot{\varepsilon} = \frac{\partial}{\partial b_m} \{+K(\frac{1}{b} \varepsilon^2)\},
\]

(19)

\[
\dot{\varepsilon} = Ke \frac{\partial \varepsilon}{\partial b_m}.
\]

(20)

Substituting (9) yields

\[
\dot{\varepsilon} = Ke \frac{\partial \psi_m}{\partial b_m}
\]

(21)

According to (11) and (12) \( \varepsilon \) is changed by changing \( K_d \), thus

\[
K_d = Ke \frac{\partial \psi_m}{\partial b_m}
\]

(22)

The gradient \( \partial \psi_m/\partial b_m \) can be determined as follows. The model equation is

\[
K_m \dot{\psi}_m = \dot{\psi}_m + a_{1m} \dot{\psi}_m + K_m' (3a_{m} \psi_m^2 + b_m \psi_m).
\]

(23)

Differentiate this equation with respect to \( b_m \)

\[
0 = \ddot{u} + a_{1m} \ddot{u} + K_m' (3a_{m} \psi_m^2 + b_m) \dot{u} + K_m' \psi_m,
\]

(24)

\[
u = \frac{\partial \psi_m}{\partial b_m}.
\]

(25)

Rewriting this equation yields

\[
-K_m' \psi_m = \ddot{u} + a_{1m} \ddot{u} + K_m' (3a_{m} \psi_m^2 + b_m) \dot{u}.
\]

(26)

Except for the non-linearity, this equation has the same form as the differential equation of the model.

The model which generates \( u \) is called the sensitivity model of the system. The sensitivity model has the same structure as the system, the input of the sensitivity model is the output signal of the reference model. When \( u \) is multiplied by \( e \) and the product is integrated, \( K_d \) is found and is used to adjust the rate feedback gain automatically.

The implementation of this adaptive control system is shown in Fig. 3.

5. LIAPUNOV APPROACH

The disadvantage of adaptation with a sensitivity model is that the resulting adaptive system is not stable under all circumstances [7]. Therefore, the synthesis of stable model reference adaptive control systems has received much attention during the past decade. However, applications are not often reported in the literature. In [13] an application based on Popov’s hyperstability theorem is described and in [14] an approach is followed in which the second method of Liapunov is used. In this section the Liapunov approach is followed.

Surveys of the Liapunov synthesis method are given by Parks [6], Hang and Parks [7] and Lindoff and Carroll [8]. In the Liapunov approach it is assumed that the system and the reference model are of the same order and that the system parameters can be adjusted directly. If there is a difference between the state variables of the model and the system, the parameters of the system are adjusted in order to minimize this difference. It should be noted that in the Liapunov approach the difference between the state variables is minimized instead of the error signal, i.e. the difference between the model and the system responses, which is minimized by using a sensitivity model. In the adaptive autopilot the Liapunov synthesis method of Gilbart et al. [9] is used which is an improvement of the method of Winsor and Roy [10], the former giving a faster error convergence. Also, simulation shows that [9] possesses better stability characteristics in a practical implementation [11].

Fig. 3. Adaptive autopilot with a sensitivity model.
In the adaptive system the transfer between $\delta_g$ and $\psi_{\omega}$ is considered under the same approximations (a–d) as in Section 3. A law will be derived for the adaptation of the rate feedback gain $K_d$.

Therefore, the differential equation of the reference model (7), which represents the desired ship dynamics, is rewritten into state space representation with $y_1 = \psi_{\omega}$

$$
\begin{align*}
\dot{y}_1 &= y_2 \\
\dot{y}_2 &= -a_{1m}y_2 - K_m'(a_m y_1^2 + b_m y_2) + K_m' \delta_g \\
\end{align*}
$$

and $y^T = (y_1, y_2)$, the superscript $T$ denoting the transpose of a vector.

Rewriting the ship dynamics equation (8) with $x_1 = \psi_{\omega}$ yields

$$
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -a_{1m} x_2 - K_m'(a_w x_1^2 + b_w x_2) + K_m' \delta_g \\
\end{align*}
$$

and $x^T = (x_1, x_2)$. The relation between the parameters of (27) and (28) is given in (12). The difference between the parameters $b_w$ and $b_m$ will be compensated by the rate feedback gain $K_d$,

$$
e = b' - b_m, \quad b' = b_w + K_d. \tag{29}
$$

The system error of the adaptive system is defined as

$$
e = y - x, \tag{30}
$$

where $e^T = (e_1, e_2)$. Because of the non-linearities in (27) and (28), the error (30) will be non-linear. However, for the application of the Liapunov synthesis method it is essential that the model and the system be linear. For the design of the adaptive loop it is assumed that linearization is allowed. Simulation has to confirm this assumption.

Subtracting (28) and (27) and neglecting the higher order terms yields the linear error equation

$$
\dot{e} = Ae + ce_1, \tag{31a}
$$

where

$$
A = \begin{bmatrix} 0 & 1 \\ -K_m' b_m & -a_{1m} \end{bmatrix}, \quad e^T = (0, \lambda), \quad \lambda = K_m' e. \tag{31b}
$$

The design procedure begins by forming a Liapunov function $V$ consisting of quadratic forms of $e$ and $\lambda$

$$
V = e^T Pe + \alpha' (\lambda + b' e)^2, \tag{32a}
$$

where

$$
v = e^T P x_1, \quad l^T = (0, 1) \tag{32b}
$$

and $P$ is a $(2 \times 2)$ symmetrical positive definite matrix with elements $p_{ij}$ and $\alpha' > 0, b' \geq 0$ are arbitrary constants to be determined later.

The time derivative of $V$ is

$$
\dot{V} = e^T (A^T P + PA) e + 2e^T P c x_1 + 2\alpha' (\lambda + b' e) (\lambda + b' e). \tag{33}
$$

The matrix $P$ can be found as the unique solution [12] of

$$
A^T P + PA = -Q, \tag{34}
$$

where $Q$ is any symmetrical positive definite matrix of appropriate dimensions and the matrix $A$ is stable.

If the adjustment equation is selected as

$$
\dot{\lambda} = -(1/\alpha') e - b' \dot{e} \tag{35}
$$

then substituting in (33) yields

$$
\dot{V} = -e^T Q e - 2b' \dot{e} \tag{36}
$$

The form $\dot{V}$ is negative definite with respect to $e$, so that asymptotic stability in the large for $e$ in the sense of Liapunov is assured, which means that after an initial disturbance $\|e\| \to 0$ for $t \to \infty$, where the Euclidean norm $\|e\| = \sqrt{(e_1^2 + e_2^2)}$.

Under the assumption that the parameter $b_w$ of (28) is varying slowly compared to the rate of adaptation $\dot{\lambda} = \dot{\lambda} = K_m' K_m'$. Substituting in (35) with $\dot{e}_1 = e$, $\dot{e}_2 = \dot{e}$ and $x_1 = \psi_{\omega}$ results in the adaptive law

$$
K_d(t) = -\alpha \int_0^t ((p_{12} e + p_{22} \dot{e}) \psi_{\omega}) d\tau - \beta (p_{12} e + p_{22} \dot{e}) \psi_{\omega} + K_d(0) \tag{37}
$$

where the constants

$$
\alpha = 1/(\alpha' K_m') > 0 \quad \text{and} \quad \beta = b'/K_m' > 0,
$$

are respectively the integral adaptation gain and the proportional adaptation gain, and they are found in simulation, but $p_{12}$ and $p_{22}$ follow from (34). This result is often referred to as the 'proportional plus integral' adaptive law.

When $\beta = 0$ the resulting adaptive law is of an 'integral' form and similar to the result which is obtained by applying [10].

The influence of setting $\beta > 0$ on the error convergence can be examined by simulation and is discussed in Section 7.

The time derivative $\dot{e}$ in (37) can be formed with a derivative circuit

$$
s \omega_c (s + \omega_d), \tag{38}
$$

where $\omega_d$ is sufficiently large. The adjustment of $K_d$ according to the adaptive law is shown in the basic diagram of Fig. 2.

When measurement noise is present in $\psi_{\omega}$, the adaptive law (37) will lead to erroneous results. Because of the noise in both $\psi_{\omega}$ and $e$, in (37) noise biasing will occur, leading to dependency of $K_d$ on the RMS value of the noise. This complication is overcome when a second order filter is used for filtering the error signal $e(t)$ as is described in the next section. Naturally, with a filter the adaptive system will not possess asymptotic stability.

The influence of the noise filter and the derivative filter (38) on the system stability can be investigated.
by linearizing the equilibrium equation and applying root-locus techniques [11], thereby providing a method for selecting a filter with the desired characteristics.

6. FILTERING

There are two types of disturbances which act upon the system.

(a) Disturbances that cause deviations from the set course.

(b) Disturbances which affect the steering characteristics of the ship, i.e. the parameters of the model.

Wind and waves belong to the first category. They should be corrected as far as possible by means of a feedback. Generally, the frequencies of the yawing motion are so high that they cannot be corrected by the rudder. Therefore, the rudder should not be activated by course deviations caused by this fast motion, as this will only give loss of speed. Usually this problem is solved by applying a dead band in the steering gear, which can be set dependent on the sea conditions. However, deviations of a lower frequency should be corrected totally, without any dead band. For this reason it is better to separate these two components with a low-pass filter, rather than using a dead band which not only damps the fast yawing motion, but also causes undesirable course deviations. The frequencies of the yawing motion should therefore be outside the bandwidth of the system.

Experiments at sea were carried out to analyze the frequency spectrum, applying test signals of several frequencies and sailing with several angles between course and the direction of the sea motion.

Afterwards the signals were analyzed using the fast Fourier transformation on the digital computer of the laboratory. The results of this analysis are shown in Fig. 6. The frequencies of the yawing are clearly separated from the frequency components caused by the rudder as shown in Fig. 6(a). In a closed loop system the high frequencies activate the rudder also as indicated in Fig. 6(b). To prevent this, trials were also carried out using a linear second-order filter, with a transfer function

$$\frac{1}{s^2/\omega_n^2 + 2\zeta/\omega_n s + 1}$$

(39)

Repeating the trials under the same circumstances as in Fig. 6(a) and (b) gave the result that is shown in Fig. 6(c) and (d). With this filter one more parameter setting of the autopilot can be omitted: the weather setting. For one particular ship the filter can have fixed settings. As the filter gives a good separation of signal and noise, the same filter can be applied in the adaptive loop. In that case the error signal between model and ship is filtered, so that
differentiation and multiplication of this signal will not produce complications. In the sensitivity model, the filter is not necessary; with the Liapunov approach, a filter in the adaptive loop is essential.

7. REALIZATION AND RESULTS

An adaptive autopilot for a ship was designed with an adaptive loop synthesized following the sensitivity model approach and the Liapunov approach, as outlined in the preceding sections. The design is shown in Fig. 2. In Fig. 3 a prototype of an autopilot using a sensitivity model is shown, which was tested at sea. In the designs the reference model was placed parallel with the rudder rate of turn transfer instead of parallel with the complete feedback loop.

This is motivated because in the reference model it is not possible to generate the heading signal electronically, as this signal is not bounded. The ship has to be able to turn several times around its axis in the same direction, which means a constantly increasing heading. Therefore, the desired heading \(\psi_p\) is also never available in an autopilot, but the course error signal only.

In laboratory studies a comparison was made between an autopilot using a sensitivity model and a Liapunov design. The parameters which are used in the simulation of the ship's characteristics are obtained from measurements at sea with the Dutch pilotship 'Capella' of 2000 brt. The data for maximum thrustpower are given in Table 1. In a first-order approximation, for small rudder-angles, the dominant time constant of the ship is 12 sec.

### Table 1. Data for Maximum Thrust Power

\[
\begin{align*}
K' &= 0.05 \\
\alpha_1 &= 3.4 \\
\alpha &= 1.06 \\
\beta &= 1.24
\end{align*}
\]

To simulate the autopilot designs, the ship's characteristics were simulated on the analogue section of a hybrid computer configuration (AD4-IBM 1800) and the reference model and the adaptive loop on the digital section. In Fig. 2 the basic design is shown, where for the computation of \(K_d\) in the sensitivity design a sensitivity filter is used as shown in Fig. 3.

To compare the autopilot designs, the performance of the adaptive loop is measured by the IAE criterion

\[
IAE = \int_0^T |e(t)| \, dt. \tag{40}
\]

The criterion is motivated because the reference model has to be followed as close as possible. Also, it is important that the ship's speed loss be minimized, which means that the applied amount of rudder is minimum, leading to the criterion

\[
IAD = \int_0^T |\delta(t)| \, dt. \tag{41}
\]
The criterion (41) can be extended with terms containing \( \psi_0 - \psi_w \), \( \dot{\psi}_w \) and the number of rudder calls. In that case the autopilot can better be called optimum adjusted than adaptive. The optimum autopilot should be used for voyages over a long distance, where it is important to save fuel and where the time is available for identification of the ship's transfer function which is needed for the computation of suitable feedback gains. The adaptive autopilot, however, is especially designed for steering in coastal waters, where the ship's behaviour is varying fast and where relatively large course alterations are necessary.

In this situation economic criteria are less important than safety and, therefore, the ship should have a fixed well-determined response on course alterations, which can be realized by an adaptive loop.

In the Liapunov design, the adaptive law (37) was implemented. The factors \( p_{12} = 1 \) and \( p_{22} = 0.3 \) follow from equation (34) where the matrix

\[
Q = \text{diag}(0.123, 0.04).
\]

The sensitivity design is implemented according to equations (22) and (26).

To obtain a sufficient damped behaviour in the autopilots the gain factors \( K_{d0} \) and \( K_p \) were selected to be \( K_{d0} = 15 \) and \( K_p = 1.5 \). Course instability was simulated by an instantaneous change of \( b \) in \( b + \Delta b \), where \( \Delta b = -5 \), at \( t = t_1 \).

In Fig. 4(a) a simulation result is shown when no adaptation is applied. Figure 4(b) shows the response of the sensitivity design, where the integral adaptation gain \( \alpha = 100 \), and Fig. 4(c) the Liapunov design response with \( \alpha = 5 \). It is seen that adaptation improves the response of the ship's heading \( \psi_w \). For a systematic comparison of the IAE criterion (40) is measured as the response for a step of the desired heading from \(-10^\circ\) to \(+10^\circ\), \( T = 70 \) sec. In Fig. 5(a) the IAE of the sensitivity design is given as a function of the speed of adaptation. Also the IAE is given when disturbances are present, represented in a Gaussian noise level \( n(t) \) with bandwidth 0.5 Hz. This noise is added to \( \psi_w \), the signal to noise ratio for the observed time interval is 7. The IAE due to noise only is also given, indicating the lower bound of noisy IAE measurements. When \( \alpha > 150 \) the response of \( \delta(t) \) is not smooth enough for practical purposes. It is seen that the sensitivity design gives good responses in the presence of noise. In Fig. 5(b) the criteria are shown for the Liapunov design. For \( \beta = 0 \) in the law (37) the IAdobe criterion (41) is given, which is fairly flat. For \( \alpha > 10 \) the responses of \( \delta(t) \) are not smooth enough, which cannot be expressed in the IAdobe itself.

Without noise the IAE comes to a low value, representing a fast response. Setting \( \beta = 5\alpha \) gives an improvement, but increases the noise level in the adaptive loop which makes this choice undesirable in a practical system. Usually \( \beta \) is selected \( \beta > \alpha \) but this value does not give a significant improvement. When noise is present, the filter (39) is applied for noise rejection with \( z = 0.5 \) and \( \omega_n = 0.55 \). The IAE is shown and it is seen that \( \beta > 0 \) does not give a significant improvement. A relative low value of \( \alpha \) gives a good performance.

Comparing the sensitivity model design and the Liapunov design it can be concluded that without noise the Liapunov design is faster. In more practical situations when noise is present, the sensitivity design gives a good performance and the Liapunov design, where a noise rejection filter is essential, is more sensitive for the setting of \( \alpha \). In general, in the noisy case the adaptation gains have to be selected low. Therefore, it can be concluded that none of the compared designs performs significantly better. This does not confirm the results of [7]; however, in [7] the influence of noise is not investigated.

Also, experiments at sea were carried out. After laboratory simulations [5] a prototype of an autopilot using a sensitivity model was built and tested on board the 'Capella'.

Because this ship is always course stable, in the experiments the initial value of the rate feedback gain was set to zero. The experiments were performed under several sets of external conditions, while the magnitude of changes in the desired heading \( \psi_0 \) and \( \alpha \) were varied. The 'best' value of \( \alpha \) was found to be dependent on the RMS value of the disturbances, i.e. the amplitude of the waves.
Generally, a high value of $\alpha$ can be exchanged for sensitivity for disturbances. Also, occasional course changes are necessary to reject drift influences in the autopilot. A typical result of sea measurements is given in Fig. 7, where $K_p = 3$, $K_{\phi 0} = 25$ were selected. The sea state was 4; no yawing filter in the rudder signal was used during this trial. The time to reach the correct value of $K_d$ is measured to be about 30 sec, which is about three times the dominant time constant of the ship.

During the experiments at sea, the yawing filter (39) was tested with $z = 0.5$ and $\omega_n = 0.55$. In Fig. 6 the frequency analysis of the measured data is given. From Fig. 6(b) it can be seen that rudder movements due to yawing are suppressed effectively.

8. CONCLUSIONS

A prototype of an adaptive autopilot using a sensitivity model was tested on board a ship of 2000 brt and produced satisfactory results when used for manoeuvring.

Occasionally course changes are necessary, because otherwise in the presence of noise the adapted parameter will drift to an incorrect value. In the absence of course changes the parameter can be set in hold mode.

Measurements indicated that the yawing frequencies were beyond the bandwidth of the ship dynamics. Therefore the high frequency rudder movements were suppressed successfully with a low-pass filter.

Simulation results of the autopilot using a sensitivity model and one designed following the Liapunov approach showed no significant difference.

In the presence of noise a low-pass filter is essential in the adaptive loop of the Liapunov autopilot.

In the near future more experiments at sea are planned, while the research in the field of adaptation is still continuing.

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